MODELS OF EARTHQUAKE FAULTS WITH LONG-RANGE STRESS TRANSFER

Seismologists, and more recently computational and condensed-matter physicists, have made extensive use of computer modeling to investigate the physics of earthquakes. Here, the authors describe a simple cellular automaton model that explores the possible relationship between Gutenberg–Richter scaling and critical phenomena.

Numerical modeling of earthquake processes has become an important proving ground for ideas that have no other experimental arena. Nearly all earthquakes originate more than 10 km underground, making seismic waves and displacements measured at the surface our only means of observation. Many details of the faulting process remain unknown, leaving us without a complete physical theory of their occurrence.

Here we discuss an approach that seeks elementary numerical models capable of reproducing some prominent features of earthquake data. Most notably, we study models that emulate the Gutenberg–Richter scaling law, which suggests the earthquake process is largely scale invariant. The presence of scale invariance coupled with the ubiquity of this law in diverse geological conditions indicates that the large-scale statistical properties of earthquakes may be insensitive to environmental details. In this case, relatively simple models will properly capture the essential physics responsible for these behaviors.

The Gutenberg–Richter scaling relation

Earthquakes primarily occur at the boundaries of rigid tectonic plates. As the plates drift, nearly all relative motion occurs across a narrow network of cracks or faults that form at the interface. Friction across the rough fault faces prevents steady sliding, instead forcing the nearby ground to deform and accumulate strain. The resulting increase in stress will eventually force the rock to fracture along a weakened fault plane. In this manner, strains accumulated at rates of a few centimeters per year over decades and centuries release in sudden shifts of several meters. The energy once stored as elastic strain is released in the form of heat, damage to the rock, and destructive seismic waves. This process is innately nonlinear, involving the interplay of viscoelastic and frictional forces, under the influence of heterogeneities of the crust, pore fluids in the rock, and innumerable other complications. There is no successful strategy for predicting when and where such ruptures will occur or how big they will grow to be.

Emerging from these seemingly insurmountable complexities is a surprisingly simple and exacting pattern: the Gutenberg–Richter (GR) scaling relation. If $N(M)$ is the number of earthquakes recorded in a given region with magnitude M or greater, we find that for a large number of recorded earthquakes a plot of $\log N(M)$ versus M reveals a straight line with slope $-w$ (see Figure 1). Because the Richter magnitude M is a logarithmic scale, $M \propto \log[E]$, where E is the energy released, we have $N(E) \propto E^{-w}$, a power law. Power laws contain an important symmetry in that rescaling by any factor λ will yield a function of the same shape, only multiplied by a constant: $N(\lambda E) = \lambda^{-w} N(E)$. Thus we say this is a *scale invariant* law: it appears

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the same in any units. With no reference to units, an earthquake of magnitude 6 appears identical to an earthquake of magnitude 1.5, even though the former releases roughly 6×10^7 times the energy.

Although data is incomplete for small quakes (as many escape detection) and rare for large ones (where fewer points make for poor statistics), there is an enormous range over which this relation holds. This suggests the physics of the earthquake process is scale invariant. This is an enormous simplification, as the same mechanisms might be at work for earthquakes of vastly different sizes. If we could understand the nature of this scale invariance, the mechanisms that produce such a vast array of earthquake sizes, we might be able to identify the conditions that lead to large earthquakes.

Fundamentally, an earthquake is a fracture, which does not develop instantaneously. A fracture initiates locally and propagates rapidly across the fault surface, eventually stopping due to dissipation or encounters with strong contact points called *asperities*. Regions far from the initiation point that participate in the rupture might be considered "correlated" in the sense that they must be sufficiently close to threshold so that elastically transferred stresses cause them to break.

Using such language, the presence of the GR scaling law is reminiscent of critical phenomena in solid-state physics, where you find a scale-invariant distribution of correlated (same-phase) regions near the threshold of a continuous phase transition. Although the analogy is not perfect, it introduces a set of powerful physical concepts that might, with some modification, apply to earthquake studies.

Chief among these concepts is the *renormalization group*, one of the major theoretical breakthroughs of the last 50 years. In a regime of scale-invariant physics, the phenomena will appear the same in any units of measure.

Given a set of data with no scale of reference, it should be impossible to determine the absolute magnitudes. However, if we chose to use large units of measure, the smaller events would become more difficult to resolve—we would lose the fine details. The renormalization group provides a procedure of coarsening the units of measure so that we eventually lose sight of the microscopic details entirely—the composition of the rock, the exact nature of the elastic forces, and so forth—while maintaining the data's large-scale features. We're left with universal behavior, depending on only a few parameters such as the dimensionality of the space. The tremendous complexity of earthquake source mechanics might give way to simple behavior at large enough scales. This is still a relatively new approach to understanding earthquakes and is an active area of study.

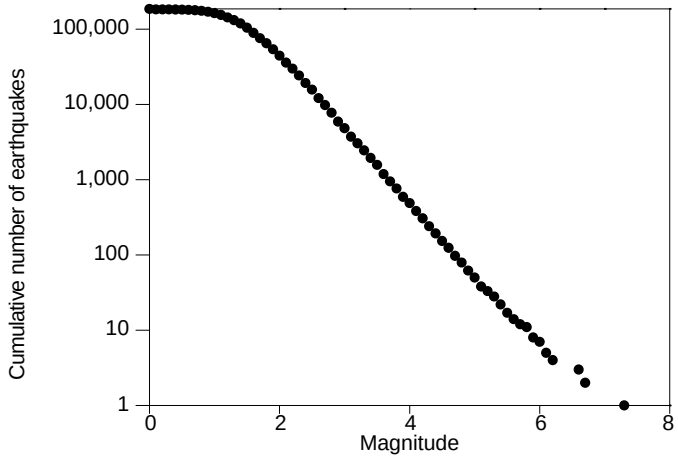


Figure 1. The cumulative number of earthquakes greater than or equal to a given magnitude in Southern California from January 1985 through August 1999 (Southern California Earthquake Center catalog). About 200,000 events are shown, one point for each unique magnitude in the catalog.

Minimalistic models

The use of minimalistic models began in 1967 when Robert Burridge and Leon Knopoff (BK) published results indicating GR-like power-law behavior from a simple chain of blocks and springs being pulled across a rough surface. In such a model, springs connect blocks representing contiguous sections of a fault to provide a linear elastic coupling. Both experimental and numerical models were constructed and tested. Computer simulation offers much more flexibility in modifying parameters and taking data and has been the primary means of investigation since.

The BK model[1] was numerically extended to 2D arrays of blocks, where directly solving the equations of motion becomes prohibitively expensive. Instead researchers attempt to approximate the dynamics with simple update rules at discrete time steps—a *cellular automaton* or CA. Such a model need only keep track of one or two real-valued fields—stress (in the model, functionally equivalent to force)[2] or slip and stress.[3] Some prefer to call these models "coupled lattice maps," reserving the name "CA" for models with integer-valued fields.

To generate earthquakes, the stress on each block is slowly stepped up to simulate tectonic loading. Blocks where the stress has reached some prespecified threshold relax instantaneously,

jumping to a point of lower energy. The resulting stress drop is distributed among other blocks within an interaction range, with some dissipation. The blocks that have received stress through this process can in turn become unstable, and the iteration of this discrete dynamic generates a cascade, or avalanche, of relaxations—the model's equivalent to an earthquake.

These models have allowed for the desired increase in size and dimensionality of the network at a fairly modest price: we have to consider slider blocks only in the over-damped regime, where their inertia plays no role. To disregard inertia implies not considering wave mechanisms for energy and stress relaxation. Because the amount of energy that wave motion carries doesn't exceed 10% of the total energy released in an earthquake, this approximation is generally considered acceptable.

The CA models also show GR-like scaling over several decades and have raised many still-open questions in the condensed-matter community. Most of the work reported so far concerns the nearest-neighbor or the random-neighbor versions of the model and addresses issues such as the role of stress redistribution or dissipation. However, neither the large-scale simulations made possible by nearest-neighbor CAs nor work done on the original BK model have succeeded in yielding a theoretical analysis that demonstrates a clear connection between the scaling and critical phenomena.

After analyzing a single BK slider block's behavior, Giovani Vasconcelos argued that a first-order phase transition occurs but that no continuous phase transition, giving rise to critical phenomena, exists.[4] The mechanism that leads to scale invariance in extended dynamical systems in general is thus still unclear. In this context, the frequently used word *critical* refers to the existence of power laws and to the absence of a characteristic scale in the system's response to the driving mechanism, not to the presence of a continuous phase transition.

Correctly addressing the physics of the earth's crust requires extending these modeling efforts. The elastic and viscoelastic interactions responsible for the crust's dynamics are long range. Although geophysicists do not know the actual expression for crustal interactions in real faults, they expect its functional dependence on distance to be roughly of the form $1/r^3$. Microcracks, as well as other heterogeneities on the fault, called *defects* in general, might include a screening effect on this coupling. The inclusion of long-range couplings such as this in a CA model can change its behavior substantially. The longer the interaction range, the more a single region or block will experience an averaged stress field, with many distant contributions smoothing over local fluctuations.

In condensed-matter physics, the extreme limit of this situation is described as *mean field*, and rigorous theoretical results are available when this condition applies. A mean-field description's range of validity is an increasing function of the interaction range—the quantitative version of this statement is known as the Ginzburg criterion, specifying the conditions under which fluctuations away from the mean field have vanishing importance.

In the mean-field regime, each block is effectively embedded in the same average stress field, with uncorrelated Gaussian fluctuations. Studies of models in the mean-field approximation have produced considerable insight into the dynamics of several different physical systems, such as glasses, charge-density waves in conductors, and magnetic hysteresis. The last two have strong similarities with our system of interest—they are nonlinear threshold systems.

In particular, a renormalization group study of magnetic domain growth with a $1/r^3$ interaction concluded that the upper critical dimension for this coupling is two;[5] this means that $1/r^3$ systems embedded in physical dimensions larger than two are effectively mean field. Thus, mean-field models might well represent real earthquake behavior better than models with only short range couplings.

A long-range CA model

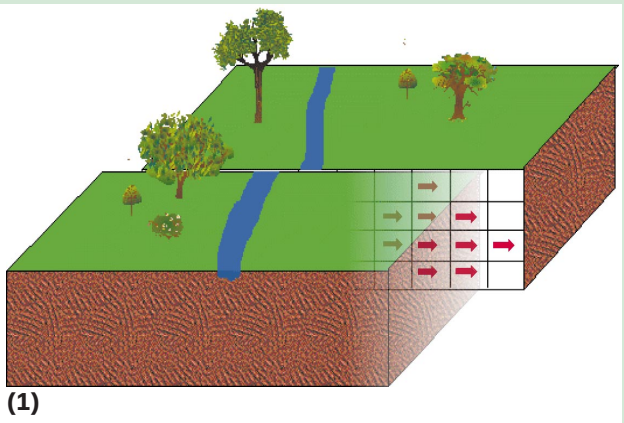
We focus here on a long-range CA version of the BK model. Earlier work done on this CA model[6] shows results for simulations with a $1/r^3$ coupling. This is not, however, the best choice if we want to examine the consequences of mean-field behavior, because this is only achieved for very large lattices (as discussed later).

The simplest model of this class has uniform interactions, where the coupling between a site and its q neighbors has a constant value, irrespective of their distance. Each site interacts with others in a square neighborhood, the distance from center to edge defining the interaction range. The coupling has to scale with $1/q$ to ensure a proper thermodynamic limit.

Simulations with long-range CAs are much more time-consuming than nearest- or random-neighbor models, and efficient coding is mandatory if you intend to simulate systems large enough to exhibit the approach to mean-field behavior.

The RJB Slider-Block Model

The cellular automaton model introduced by John Rundle, David Jackson, and Steven Brown consists of a 2D array of massless blocks that interact with their neighbors and a tectonic loader plate through linear springs with constants $K_C(r)$ and K_L, where r is the distance between blocks (the labels 'c' and 'l' refer to 'coupling' and 'leaf' springs used to construct the original models). Each block represents a discrete patch of a fault plane (see Figure A1), which is assigned a single collective position coordinate U_i. The elastic forces between the patches are taken to be linear, so that you can visualize the model as a collection of blocks and springs (for example, see the article in this issue by Bruce Malamud and Donald Turcotte). Using long-range interactions, you must imagine a network where every block is connected to every other block with some sort of spring, not just nearest neighbors. An earthquake consists of one or more of these patches slipping relative to one side of the fault taken as fixed. Slipped sites redistribute stresses to other blocks in the system, resulting in an avalanche. Figure A2 shows a typical event for a lattice of 128×32 sites, hotter colors indicating more slip.



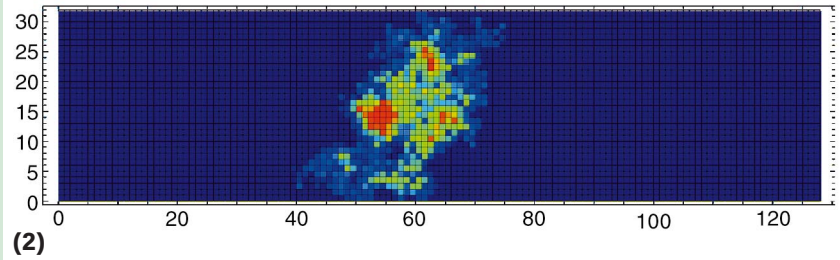


Figure A. To simulate an earthquake, the fault surface is (1) partitioned into discrete patches which stick or slip as units. The regions are coupled to simulate the elastic forces in the crust that transfer stress as sites slip. (2) An example event from a 128 × 32 array shows the tapering of slip near the edges as transferred stress is absorbed and dissipated. Red regions indicate large slips.

Initially, each block i is assigned a random position U_i from a uniform distribution. We initially set the loader plate contribution to the stress equal to zero and calculate the stress σ_i on each block with

$$\sigma_i(t) = \sum_{j \neq i} T_{ij}\left(U_j(t) - U_i(t)\right) - K_L U_i(t) = \sum_j T_{ij} U_j(t) \qquad \text{(A)}$$

This stress is compared to a threshold value σ_i^F. T_{ij} is a matrix of interaction coefficients—$K_C(r)$, for a translationally invariant system. The self-interaction is defined as $T_{ii} = -K_i$, where K_i is the effective spring constant $K_i = K_L + \sum_{j \neq i} T_{ij}$. The sum over $j \neq i$ includes all of the neighbors of block i, while the sum over j includes also the block i itself. If $\sigma_i < \sigma_i^F$ then the block is not moved. If, however, $\sigma_i \geq \sigma_i^F$ the block is moved

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Algorithms for CAs with long-range interactions

Simulating a long-range CA model presents great challenges. The active faults whose behavior the model addresses have depths on the order of a kilometer or more. Because our slider block models a macroscopic fault asperity, which has linear dimensions on the order of 10 meters, the linear size of the model's lattice should be on the order of 100 blocks or larger. Efficient coding of nearest-neighbors BK-based CA models is discussed in the literature, allowing the simulation of lattices of linear size on the order of 1,000,[7] but some of the techniques therein are not suited for long-range interactions and require some adaptation.

We devised some basic strategies when simulating the single-field version of the RJB (Rundle, Jackson, and Brown) model with long-range interactions with a cutoff in the zero-velocity limit (for a detailed description of the RJB model, see the sidebar).

A naive transcription of this model's dynamic rules involves unnecessary lattice sweeps. Our algorithm's goal is to keep these lattice sweeps to a minimum. The search for the initiator of the next event—the site nearest to failure—is in this case unavoidable. Keeping the sites in separate bins according to their stress and searching only the bins with the largest stresses—a strategy that pays off for the nearest-neighbors version—does not work in this case.

For long-range interactions, a site that fails must redistribute its stress to a large number of neighbors, and constantly resorting the sites into separate bins is too costly in CPU time to be of any use.

according to the rule

$$U_i(t+1) = U_i(t) + \left[\frac{\sigma_i(t) - \sigma_i^R}{K_i}\right](1 - \eta_i(t))\Theta\left(\sigma_i(t) - \sigma_i^F\right). \quad \text{(B)}$$

Here, the step function is $\Theta(x) = \{0\ \forall x < 0; 1\ \forall x \geq 0\}$. This mathematically expresses the Mohr–Coulomb friction law, which has the useful property of being scale invariant. It is also the source for the model's nonlinearity and embodies a typical form of threshold dynamics. The residual stress σ_i^R is a parameter that specifies the stress on a block immediately after a noiseless system fails.

In this article, we discuss the stochastic CA model, in which $\eta_i(t)$ is a random noise equal to ρW, where ρ is a uniformly distributed random number $\in$ [0, 1], and the predetermined noise amplitude W is in the range $0 \leq W \leq 1$. We measure the discrete time t in Equation B in the relaxation scale, and we assume that the long-range interactions can be considered as instantaneous on this scale. We justify using this approximation here because the physics of interest is the development of space–time correlations between blocks in the system, which evolve on typical time scales much longer than the source time scales characterizing the relaxation. Such long-time correlations are the only time scales of interest for critical phenomena, which are the underlying physics related to the observation of the scaling laws.

You can translate our description of the coupling between blocks into an effective global coupling of the slip and stress fields at times $t + 1$ and t. In practical terms, you have to compute the values of these fields at every site in the lattice at time $t + 1$ using its values at time t; this is called a *parallel updating of the lattice*. All blocks are tested and moved until no block has a stress greater than σ_i^F. The amount of time in the relaxation scale needed for this to happen will be called the *duration of an event*.

At the beginning of each successive time step, each coordinate is incremented slightly to simulate tectonic loading. We can do this with two different procedures. The first, which we refer to as the *zero-velocity limit*, requires finding the block that is closest to failure—that has the smallest value for $\sigma_i^F - \sigma_i$. We then add to each coordinate so that each block receives that amount of stress,

$$\sigma_i \Rightarrow \sigma_i + \Delta\sigma = \sigma_i + \left(\sigma_i^F - \sigma_i\right)_{min} \quad \text{(C)}$$

which will bring the former block to its failure threshold. This guarantees that in the vast majority of plate updates, there will be only one initial failure site and hence one earthquake per plate update. We obtained all the results shown here with this loading procedure. The second method is to increment the coordinates a fixed distance $V\Delta T$, thereby increasing the stress σ_i on each block i by $K_L V\Delta T$. The quantity ΔT determines the tectonic time scale. After the plate moves, we calculate the stress on each block through

$$\sigma_i(t) = \sum_j T_{ij} U_j(t) + K_L V \sum_n \Theta(n - t) \quad \text{(D)}$$

where the summation over n essentially counts the number of plate updates since the beginning of the simulation. Then we repeat the relaxation process outlined earlier.

For both methods of plate updating, we assume that the tectonic time scale is much larger than the relaxation time scale, providing a complete separation between the loading and failure processes. The space–time correlations between blocks, induced by the dynamics, occur over the tectonic time scale. Separation of these time scales is a well-justified assumption when you consider the dynamics of real strike-slip faults, where the ratio of typical speeds of loading to relaxation durations is larger than 10^{10}. We thus have a coupled relaxation and an uncoupled loading dynamics. This time–scale separation is made radical in the zero-velocity limit, which is why we omitted a notation for time in Equation C. Nevertheless, we can still compare time between events through $\Delta\sigma$.

The model also admits a single-field formulation, by eliminating the slip field $U_i(t)$. The RJB dynamics permit a much more efficient coding when cast into this form. Although we

On the other hand, in mean-field models you can avoid updating stresses throughout the lattice after a plate movement. Instead, the code uses a moving threshold, which creeps downward as stress accumulates from driving or from site failures.

For lattices with finite interaction ranges, we can eliminate lattice sweeps by only iterating over blocks within the interaction range of failed blocks. This requires keeping lists of these blocks in auxiliary vectors.

All the structures in the code are 1D. The 2D lattice is mapped into a vector, with a side effect of decreasing the space needed for the auxiliary lists—they store a single number, the address of a site, instead of a pair. This mapping also significantly decreases CPU time; in the worst case, it does exactly the same thing a smart compiler would do, because all internal structures in computer memory are in reality 1D.

Simulation results

Because of its simplicity, we can use the long-range RJB model to test a number of theoretical ideas. As an example, we introduced some subthreshold dynamics on the stress field.

The effective range of interaction R is proportional to the square root of the second moment of the coupling Green's function $R^2 \sim \int_0^C r^2 K_C(r) d^D r$, where C in the upper limit of the integral is the cutoff. For the uniform interaction, R is propor-

retain knowledge of correlations in the stress field, the price we pay is losing track of interesting correlations in the slip field. We avoid subtle considerations about the duplicity of time scales by specializing to the zero-velocity limit and by focusing on the relaxational dynamics, because the loading process is already cast into a form where it depends solely on the stress field. To accomplish this simplification, multiply Equation B by T_{ij}, sum over j, and use Equation A to obtain

$$\sigma_i(t+1) - \sigma_i(t) = \frac{1}{K}\sum_j T_{ij}\,\frac{\sigma_j(t)-\sigma_j^R}{K_j}\,\Theta\left(\sigma_j(t)-\sigma_j^F\right) + \eta_i'(t) \qquad \text{(E)}$$

where the noise

$$\eta_i'(t) = -\sum_j T_{ij}\,\frac{\sigma_j(t)-\sigma_j^F}{K_j}\,\eta_j(t)\,\Theta\left(\sigma_j(t)-\sigma_j^F\right)$$

remains spatially random as long as T_{ij} is a radially symmetric interaction (which we assume).

Equation E gives the new stress on block i at time $t + 1$ in terms of the previous stresses from block i, the neighboring j blocks, and the noise. From this equation, we can immediately see that apart from noise, after block j slips, each neighbor i of the failed block receives an amount of stress equal to $T_{ij}(\sigma_j(t) - \sigma_j^R)/K_j$, while the system dissipates the amount $K_L(\sigma_j(t)-\sigma_j^R)/K_j$. This stress transfer can cause these neighbors to slip, thus initiating an earthquake, which continues until every block has $\sigma_i < \sigma_i^F$. An earthquake's size equals the number of slipped blocks after each plate update. If we count each slip as a block failure, even if a block slips more than once, the resulting failures can be shown to be proportional to the event's moment. We can also compute the event's area by counting at most one slip per block. Multiple slips of a single block during one event are rare. For simplicity, we chose the threshold and residual stresses to be uniform throughout the lattice.

It is the properties of the earthquakes or avalanches in this model that we have studied both theoretically and through simulations as discussed in the main text. It is well known from the study of similar driven-threshold systems in many different contexts that the boundary conditions used can play a significant role in their overall behavior. It is usual to consider three classes of boundary conditions:

• *Periodic*: The lattice wraps around, so that sites at the right-most column are nearest neighbors to sites at the left-most column, similarly for the top and bottom. All failing sites interact with the same number of neighbors, irrespective of their position. With these boundary conditions, the lattice boundary does not introduce any inhomogeneity. This is the most common choice in condensed matter physics, because it eliminates the surface contribution to the free energy and makes it sensible to address the thermodynamic limit even with finite-size systems. For the RJB model, this choice turns the self-interaction coupling into a translationally invariant quantity ($K_i = K$ for every i). Its applicability to earthquake fault modeling is nevertheless questionable, because we expect surface inhomogeneities to play a significant role.

• *Closed*: Sites near the boundary have fewer neighbors, and, for that reason, have a smaller value for the self-interaction K_i. Failing sites at the boundary cause larger dissipation than those inside the bulk; their stress drop minus the amount dissipated is redistributed among fewer neighbors. The system loses translation invariance and has a strong source of inhomogeneity at its boundaries.

• *Open*: sites near the boundary have "ghost" neighbors outside The lattice that never slip, regardless of the stress they receive. The self-interaction and dissipation are again translation invariant, as in the first case, but the boundaries cause inhomogeneities, because the "real" sites never receive stress transfer from the ghost sites.

Strictly speaking, none of these boundary conditions reflect with fidelity the natural conditions of a real active fault, and the results that depend on the particular choice made must be considered with care.

tional to C in any dimension D. The same is not true for the inverse cubic interaction $1/r^3$, where in 2D the range is proportional to $C^{1/2}$. For this reason, using $1/r^3$ interactions requires larger cutoffs (and larger lattices) to observe mean-field behavior in simulations. Consequently, we chose to use a model with uniform interactions K/q.

We begin with a scaling plot for the frequency distribution of events as a function of area. Figure 2 shows a typical result for a 256×256 square lattice—the scaling region extends over more than three decades. For all three boundary conditions examined, a power law fit of the form $f(x) \sim x^{-\tau}$ gives an exponent τ close to the value 3/2 predicted by mean-field theory. This exponent increases from its value in the nearest-neighbors version as we increase the range of the interaction until it converges to the above value. The cutoff needed for this convergence might vary with the particular boundary condition used. As far as the scaling is concerned, the mean-field regime is reached for a range of interaction much smaller than the lattice linear size, and we will assume that this is also true in general.

The frequency distribution of the event duration, defined as the number of (virtual) lattice sweeps needed for complete relaxation after an initial site failure, also shows scaling, with an exponent consistent with the mean-field prediction of 2.0. We also examined the frequency dis-

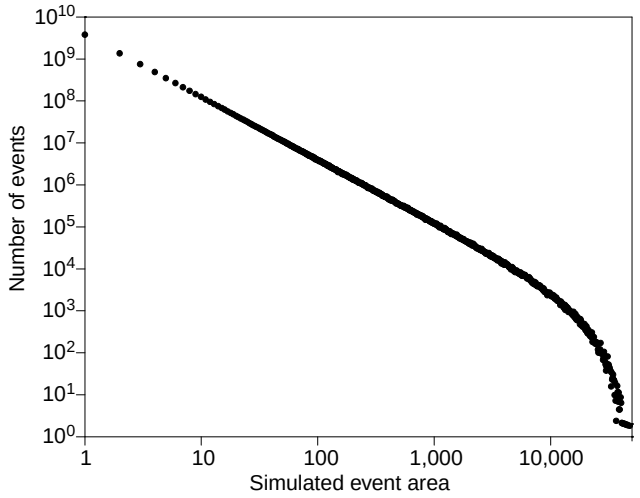

Figure 2. A log–log plot of the number of simulated events of a given area or number of blocks (instead of plotting the log of a quantity, we may logarithmically scale the axis to the same effect). Earthquake magnitude is proportional to the log energy that scales with area, so this plot is the artificial equivalent of Figure 1. The scaling regime extends to the smallest events, because detection is not a problem in simulation.

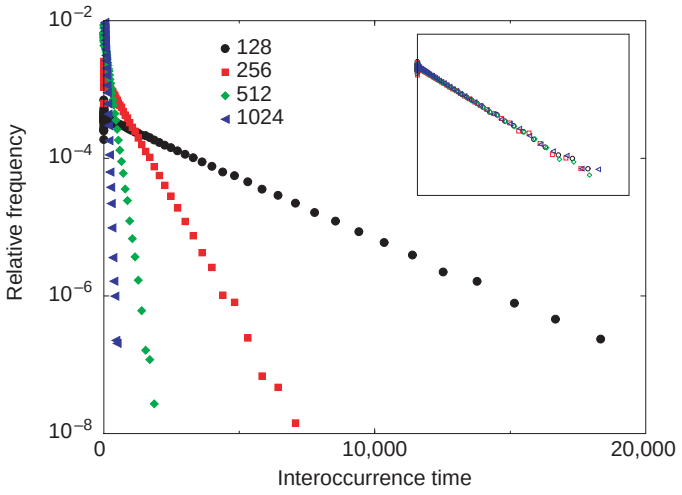

Figure 3. Distribution of time spans between events for different lattice sizes. The exponential decrease in time is characteristic of a Poisson process. The characteristic time between events scales with the number of sites in the lattice. When rescaled to the lattice size, all the distributions become identical (inset).

tribution of interoccurrence time, defined as the time in the tectonic loading scale between two successive events, irrespective of their sizes.

Figure 3 shows the results of simulations with different-sized lattices, in a semilog plot. The observation of an exponential decrease with time $f(t) \sim e^{-t/tc}$ is a clear signature of a Poisson process, in which the probability density of the occurrence of an event is time-independent $p(t) = 1/t_c$. The characteristic time t_c is the expectation value for the time between two events.

The distribution of stresses in the lattice following an event is roughly uniform, implying a scaling of t_c with the number of sites in the lattice L^2. This scaling is verified in our simulations, as Figure 3 shows. The interocccurrence frequency distributions become identical for all lattice sizes if t_c is scaled with L^2 and the value of the distribution with L^{-2} accordingly. If, on the other hand, we restrict our observations to events larger than a certain size, the frequency distribution loses its Poisson character and begins to peak around some value for interocccurrence time.

Recent developments

The single-field formulation of the long-range RJB model was recently the subject of a theoretical approach that could indicate the presence of a spinodal critical point, with properties similar to a critical point for a continuous phase transition.[8] This approach was based on a coarse graining of the model based on assumptions that simulation results support. This spinodal-critical point can give rise to the GR scaling observed in this class of models.

Identifying spinodal scaling also provides a means to classify events in the model, where four separate earthquake orders of magnitude can relate to different generating mechanisms.[9] Events that include only sites that were very close to threshold are called *fundamental clusters*. These events occur at correlated regions of high stress associated with critical fluctuations near the spinodal. Their scaling reproduces with great fidelity the scaling for event totality, with the predicted exponent of –1.5. The second class of events represents the coalescence of fundamental clusters into larger structures.

Nearer the spinodal point, the theory predicts that a different phase of globally higher stress becomes the preferred state. We expect the high-stress fluctuations to grow and take the whole system into this new phase. It is believed that such a process does occur in the models, but a mechanism arises that restricts the growth of the high-stress regions. Eventually, a site in the region will reach threshold, and the entire area will rupture and decay back to the low-stress phase, moving the system farther from the global spinodal. The large events associated with these regions of arrested nucleation also exhibit scaling, but with exponent 2.0.

The fourth class consists of events that manage to propagate past regions of high stress, causing even blocks initially far from threshold

to slip. These "breakout" events are the largest, involving nearly every site of the lattice, and do not scale on a single fault.

Simulation results are consistent with this theory, showing a strong relationship between earthquakes in the model and behavior near a mean-field spinodal. They support previous claims that techniques of equilibrium statistical mechanics can describe this model's mean-field version. The simulations also demonstrate locally ergodic behavior in the limit of zero tectonic velocity.[9]

Another interesting development addresses mechanisms for smoothing the stress field. One such proposal postulates a creeping interevent dynamic in the form

$$\frac{dU_i(t)}{dt} = f\,\frac{\sigma_i(t) - \sigma_i^R}{K_i}.$$

This lets blocks creep slightly between events but not enough to entirely relieve the building stress. For the CA model, with its infinite range, uniform interactions, and uniform residual stress, this preslip dynamics has an analytical solution. The stress field's variance has an exponential decrease between events, with a rate controlled by the parameter *f*. This smoothing of the stress field would in turn favor the occurrence of larger events. We introduced a simple linearization of this dynamic into simulations, and the results clearly confirm this idea. As *f* increases, the GR scaling deforms and large system-wide events become more frequent. Event duration distribution is also affected: a broad peak appears for large durations, indicating the possible presence of a large correlated cluster inside the lattice. The distribution of interoccurrence time loses its Poisson character, and a clear clustering in time of large events appears.

All these features have correspondences with what we observe in real faults, and these results might indicate that some sort of preslip dynamics is the missing link between earthquake modeling and real-world data.

The use of simple models, amenable to making the best use of today's computing power as well as yielding to analytical treatment, is without a doubt a powerful approach to difficult nonlinear problems. Without a clear understanding of the basic physics, we can easily misinterpret results from more complex models. Knowledge gained from analyzing the simple CA models discussed here can help us construct more general and realistic earthquake models in the future.

Acknowledgments

We make no claim that our summary is comprehensive. As a consequence, we would like to beg the indulgence of our colleagues whose work we do not have the space to discuss, and we refer the interested reader to the literature for summaries of this rapidly increasing body of research.

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