

## Simulated ecology-driven sympatric speciation

J. S. Sá Martins,<sup>1</sup> S. Moss de Oliveira,<sup>2</sup> and G. A. de Medeiros<sup>2</sup>

<sup>1</sup>Colorado Center for Chaos and Complexity, CIRES, CB 216, University of Colorado, Boulder, Colorado 80309

<sup>2</sup>Instituto de Física, Universidade Federal Fluminense, Avenida Litorânea s/n, Boa Viagem, Niterói, 24210-340, Rio de Janeiro, Brazil

(Received 29 November 2000; revised manuscript received 15 February 2001; published 20 July 2001)

We introduce a multilocus genetically acquired phenotype, submitted to mutations and with selective value, in an age-structured model for biological aging. This phenotype describes a single-trait effect of the environment on an individual, and we study the resulting distribution of this trait among the population. In particular, our simulations show that the appearance of a double phenotypic attractor in the ecology induces the emergence of a stable polymorphism, as observed in the Galapagos finches. In the presence of this polymorphism, the simulations generate short-term speciation, when mating preferences are also allowed to suffer mutations and acquire selective value.

DOI: 10.1103/PhysRevE.64.021906

PACS number(s): 87.23.Cc, 07.05.Tp

### I. INTRODUCTION

The theory of evolution studies processes that are extremely complex, and whose characteristic times usually extend over many human life spans. Because of this inherent complexity, these studies do not lend themselves readily to the establishment of discrete, falsifiable hypotheses. For this reason, observations and experimental results emanating from evolutionary studies often elicit broadly differing interpretations [1]. A large number of mathematical models has been applied to a variety of evolutionary problems, as well as to the related fields of population dynamics and evolutionary ecology, composing a unified approach under the name of evolutionary population ecology [2]. The success of many of these models, together with the above-mentioned difficulties in the interpretation of observational data, has triggered the development of an integrated computational effort for the study of these problems. Computer simulations of the behavior of virtual populations, immersed in well-defined habitats and evolving under a representation of the dynamics of evolution, have generated a wealth of reliable data with which to test our models and the assumptions they rely on.

Our approach in this paper is to use a computational representation to study the genetic patterns generated by the order-disorder conflict between selective pressure and mutation accumulation in the presence of an environment that favors particular phenotype configurations. Our model is a variation on the popular Penna model [3], a bit-string representation of genetic dynamics for age-structured populations that has been successfully used to address a variety of problems in the field. It provides a simple metaphor for evolutionary dynamics in terms of the mutation accumulation theory. In essence, each individual is represented by a double strand of inherited genetic material, subject to the addition of harmful mutations at birth. Its lifespan is determined by the position of the active deleterious mutations in the genome, while nonmutated genes have no effect on the individual adaptation to its environment.

To this underlying age-structured genome we add a non-structured portion, which can be seen as representing genes that do not have an age-dependent action. The configuration of this portion of the genome is translated into a phenotype

which determines, through its match with an environment-dependent ideal, the probability of survival when the number of harmful mutations is not critical. Models with similar characteristics were studied recently, but under a different perspective [4]. In our case, the phenotype is translated into a fitness function merely by counting the number  $n$  of positions in the bit string that are set to a given value, say 1, and the individual fitness is a function of this number  $f(n)$ . This strategy simulates a single phenotypic trait which is encoded in a number of genes. We then study the distribution of phenotypes of the equilibrium population, identifying its main patterns.

Beyond the theoretical interest of unveiling the features generated by evolutionary dynamics on the phenotype of the population, our model also allows us to address a much more challenging problem, namely, that of sympatric speciation. Speciation involves the division of a species on an adaptive peak, so that each part moves onto a new adaptive peak without either one going against the upward force of natural selection. This process is readily envisioned if a species becomes subdivided by a physical barrier, whereby each part experiences different mutations, population fluctuations, and selective forces, in what is called the allopatric model of speciation. In contrast, conceiving the division of a single population and radiation onto separate peaks without geographical isolation, in what is called sympatric speciation, is intuitively more difficult [1]. Through which mechanism can a single population of interbreeding organisms be converted into two reproductively isolated segments in the absence of spatial barriers or hindrances to gene exchange?

As an answer to this question, a general mechanism for sympatric speciation was modeled with a simple one-locus two-allele dynamics that shows how a stable polymorphism can exist in a heterogeneous environment with two niches, even when the adults form a single randomly mating population [5]. Increasingly realistic models that explicitly incorporate density-dependent competition for resources among phenotypes since appeared in the literature [6]. The present paper is another link in this chain. We show that a stable polymorphism is generated when the environment switches from a one- to a two-niche configuration in a model with age-structured genomes and competition among phenotypes,

as a necessary result of the dynamics of evolution.

The establishment of a stable polymorphism is the first step toward sympatric speciation. The second step is the subsequent evolution of reproductive isolation between populations in the two niches. The key question here is whether a single gene difference could produce selective coefficients large enough to drive reproductive isolation. It is generically thought that this isolation requires a genetic association between traits, conferring adaptation to a niche and traits causing mating preferences [7]. Alternatively, one can consider an assortative mating of phenotypes; since a gene for assortative mating can increase to fixation sympatrically, speciation could be the outcome of sexual selection alone. This is what our model shows: once we introduce a gene which describes the mating preferences of an individual, it is led by the dynamics to a fixation pattern in which alike phenotypes mate exclusively.

As a metaphor for the results obtained in our simulations, we compare them qualitatively with the observed dynamics of the Galapagos finches. Growing evidence for the onset of stable polymorphism in nature comes from years of observations of seasonal morphological variations in the population of ground finches in the Galapagos Islands. The assumptions that selection, mediated through rainfall and its effects on food supply, can have a dramatic impact on finch phenotypes, and that much of the morphological variation in these birds is genetically inherited, are reasonably well established since the field work of Grant and co-workers [8–10].

## II. PENNA MODEL WITH PHENOTYPE SELECTION

In this section we will briefly describe the main characteristics of the standard age-structured Penna model, together with the features we added to it to represent phenotypic selection. For a recent review on the applications of the model, we direct the reader to Ref. [11] and references therein.

For sexual populations, the genome of each organism is represented by two computer words. In each word, a bit set to one at a locus corresponds to a mutated deleterious allele; a “perfect” strand would be composed solely of zeros. The effect of this mutation may be felt by the individual at all ages equal to or above the numerical order of that locus in the word. As an example, a bit set to 1 at the second position of one of the bit strings means that a harmful effect may become present in the life history of the organism to which it corresponds after it has lived for two time periods. The diploid character of the genome provides for a coverage against the effectiveness of these harmful mutations. Thus a mutation in a position of one of the strands is felt as harmful only if this is a homozygote locus or if the mutated allele is dominant. In a homozygote locus, a harmful mutation must be present in both strings at the same position to be effective. The concept of dominance, on the other hand, relates to loci in the genome in which a harmful mutation in just one strand is enough to make it affect the organism’s life. The lifespan of an individual is controlled by the number of effective mutations active at any instant in time. This number must be smaller than a specified threshold to keep the individual alive; it dies as soon as this limit is reached.

An individual may also die because of intraspecific competition for the finite resources of the environment, or because of the action of predators. In the standard model, these constraints are taken care of by the so-called Verhulst factor. This is a logistic-type term that introduces a mean-field random death probability, independent of the quality of the genome, and in the simulations its main purpose is to limit the size of the population. Its usual expression is  $V(P) = P/C$ , where  $P$  is the total population at some time step, and  $C$  is a parameter of the simulation, traditionally called the carrying capacity of the environment. This name, inherited from simple logistic models [2], is somewhat misleading in the context of more sophisticated models; in those simple models the equilibrium population ends up to be  $C$ , thus justifying its name. This is not the case for models that include a more detailed microscopic description of the evolutionary interactions, where the equilibrium population is a complex function of many different parameters.

Since random deaths in nature can hardly play any significant role in population dynamics, this concept has already been criticized in the literature, and a couple of alternate implementations analyzed [12]. Rather than random, one should expect that deaths caused either by predation or because of intraspecific competition should also have selective values. The probability of their occurrence should be dependent on the fitness of the individual to the environment, and this fitness should be a function of the match between the individual’s phenotypic expression of genetically acquired traits and a phenotype ideally adapted to its habitat. The problems that such a microscopic description of selective dynamics entail are extremely complex, and are far from being solved. The present paper reports on an attempt to address this issue in the context of an age-structured model. In addition to the usual double strand of genes with an age-dependent expression, we introduce an extra pair for each individual in the population. In this computer representation, each genome may be considered as composed of 64 loci—in a 32-bit word machine—of which half have age-dependent expressions. The dynamics of reproduction and mutations, to be described in what follows, are the same for both the age-structured and new strings. Meiosis, phenotypic expression controlled by homozygosity or dominance, and the introduction of random mutations at birth equally affect all loci. The new portion of the phenotype, however, represents some individual trait, such as the size of the beak in a ground finch, for example, that may have a selective value. We chose a multiloci representation of a single trait for simplicity, and claim that there is no loss of generality in this choice for our present purposes. The selective value of this phenotype is expressed by a fitness function  $f(n)$ , chosen to depend on a single variable  $n$ : the number of 1’s expressed in the non-structured portion of the phenotype, either because of homozygosity (11 alleles) or the presence of 10 (01) configuration in a locus where the 1 allele is dominant. This fitness function encapsulates the selective value of a particular phenotype, and restricts the multidimensional space of the interaction between the individual and its environment to one single dimension. In this discretized mathematical model, the probability of death by intraspecific competition at each time

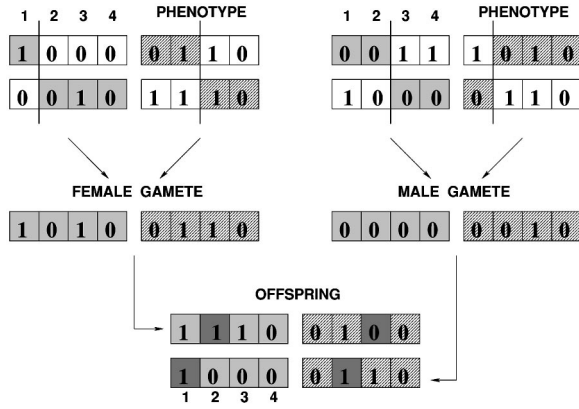


FIG. 1. Schematical representation of the reproductive process. The age-structured part of the gamete has a light-shaded background, while the nonstructured part is shown over diagonal stripes. The dark-shaded squares of the offspring genome correspond to the positions where new mutations were added.

step is now given by  $V(P)/f(n)$ . The interaction with other individuals is still mediated by the mean field  $V(P)$ , but its effect is no longer uniform. It depends on the particular genetically acquired configuration of each individual and, although stochastic, escapes from the biologically unmotivated randomness mentioned above.

Reproduction is modeled by the introduction of new genomes in the population. Each female becomes reproductive after having reached a minimum age, after which she generates a fixed number of offspring at the completion of each period of life. The meiotic cycle is represented by the generation of a single-stranded cell out of the diploid genome, now composed of two functionally different portions. To do so, each string of the parent genome is cut at a randomly selected position, the same for both strings and independently for each functional part, and the left part of one is combined with the right part of the other, thus generating two new combinations of the original genes, see Fig. 1. The selection of one of these completes the formation of the haploid gamete coming from the mother. For mating, a male is randomly selected from the population, and undergoes the same meiotic cycle, generating a second haploid gamete out of his genome. The two gametes, one from each parent, are now combined to form the genome of the offspring. Each of its strands was formed out of a different set of genes. The next stage of the reproduction process is the introduction of  $M_a$  independent mutations in the newly generated genetic strands of the age-structured part of the genome. In this kind of model it is normal to consider only the possibility of harmful mutations, because of their overwhelming majority in nature, and this is what is done for the age-structured portion. For the nonstructured portion, mutations are added independently of their harmful or beneficial effect, with some probability  $M_p$  per string. The gender of the newborn is then randomly selected, with an equal probability for each sex. Figure 1 is a schematical representation of the entire reproductive process.

The passage of time is represented by the reading of a new locus in the age-structured portion of the genome of

each individual in the population, and the increase of its age by 1. After having accounted for the selection pressure of a limiting number of effective harmful mutations and the stochastic action of the fitness function, females that have reached the minimum age for reproduction generate a fixed number of offspring. The simulation runs for a prespecified number of time steps, at the end of which averages are taken over the population. Typically, measures are taken for the age structure of the population, such as the number of individuals and probability of survival and death by genetic causes for each age group, as well as for the genetic composition distribution. We are particularly interested in the equilibrium distribution of the phenotypes in the population, also averaged at the end of each run.

### III. SIMULATION RESULTS

We will relate our results qualitatively to the observations cited in the introduction made on the evolutionary studies of the Galapagos finches. Thus we will have in mind a picture of our population as composed of ground finches, feeding primarily on seeds. The phenotype trait represented by our model describes the size and strength of their beaks, and the fitness measures their adequacy to the kinds and sizes of seeds available in the ecology.

Let us start with results concerning the equilibrium configuration of the single trait that characterizes the fitness of an individual's phenotype. For this study, the fitness function is

$$f(n) = 1 - \frac{32-n}{A}; \quad (3.1)$$

$n$  is the number of bits set to 1 in an individual's phenotype, and  $A$  is a boost factor that can be used to control the intensity of the selective pressure; this intensity is a decreasing function of  $A$ . This fitness function expresses an environment where the ideal phenotype would be composed entirely of bits 1. For instance, it can represent an environment in which the availability of edible seeds is a decreasing function of their size, while the number  $n$  is a measure of the size of the beak of an individual. In this ecology, a well-fit individual would have a large beak, allowing him to take advantage of the distribution of seeds size. The simulation starts with an ill-fit population, in which all the individuals have the smallest beak possible; in our representation, their phenotypes are homozygotes 00 in all loci. Alternatively, this can be expressed as a distribution of beak sizes  $\delta$ -like peaked at  $n = 0$ . Figure 2 shows the equilibrium distribution reached after 50 000 Monte Carlo steps, for a weak selective pressure, corresponding to a broad distribution of seed sizes, and, in our representation, to a large value of the boost  $A = 128$ . We plot the distribution of beak sizes for various values for the number of loci  $d$  in which the well-fit (1) allele is dominant in the phenotype. In all cases, the distribution shows a bell-shaped Gaussian-like aspect in which the peak position and amplitude increase with  $d$ , while the width shows a decreasing pattern. This result is easy to understand: because of the availability of large seeds, represented here by a fitness func-

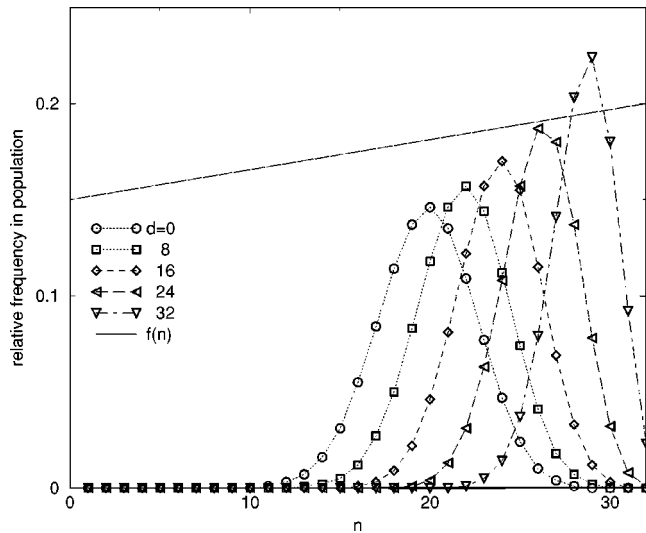


FIG. 2. Distribution of the beak size and strength, measured by the dimensionless number  $n$  of 1 alleles in the phenotype, over the population. Results are shown for various values of  $d$ , the number of loci where the 1 allele is dominant. As  $d$  increases, so does the mean value of  $n$ , whereas the width of the distribution decreases. When  $d=32$ , and the 1 allele is dominant in all loci, the distribution approaches a narrow bell-shaped curve peaking at a value close to 32. Also shown is the fitness function used in the runs, rescaled by a constant. All runs were 50 000 Monte Carlo time steps long, with a boost  $A=128$ , representing a weak selection. Other relevant parameters were 400 000 for  $C$ , the conventional carrying capacity of the standard Penna model,  $M_a=1$ , and  $M_p=0.01$ .

tion that increases with  $n$ , the population is driven to a pattern of larger beaks. The combined effects of mutation pressure and heterozygote avoids its collapse into a  $\delta$ -like distribution from which it evolved; the width of the distribution increases with the mutation rate. In a strongly selective ecology, represented by a smaller value of  $A$ , the pattern is essentially the same. The peaks are shifted to larger values of  $n$  and the widths decrease.

The shift in the distributions caused by stronger selection is shown in Fig. 3. We compare results obtained for a fixed value of  $d=0$ , where allele 1 is recessive in all loci. When there is no activation of phenotype selection, which can be translated into a uniform distribution of seed sizes, the broad pattern of beak sizes is a result of random genetic drift. If  $d=16$ , the equilibrium distribution peaks at  $n=16$ . When phenotype selection is activated, the equilibrium distribution tries to follow the availability of seed sizes, and its peak moves to larger values of  $n$ , while its width shrinks. It is worthwhile to remark that even in a very strong selective ecology ( $A=33$ ), the distribution still retains some broadness, representing a diversity of morphologies also observed in real populations.

We examine next the pattern of homozygote and heterozygote generated in the phenotype of the population. Figure 4 was obtained for a medium range selective pressure  $A=48$ . The circles indicate what is the dominant allele in each locus. One can immediately identify that the selection pressure pushes the loci in which the allele 0 is dominant to

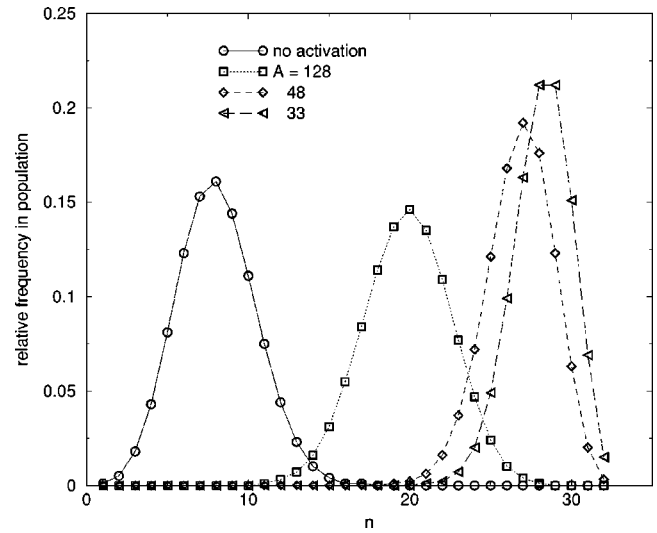


FIG. 3. Distribution of the beak size and strength, measured by the dimensionless number  $n$  of 1 alleles in the phenotype, over the population. Results are shown for various values of  $A$ , representing the intensity of the selection pressure, for a fixed value of  $d=0$ , meaning that the allele 1 is recessive in all loci. The shift of the distribution to larger values of  $n$  when the selection pressure becomes stronger is clear. The parameters of the simulations were the same as in Fig. 2

homozygote 11, while on sites where allele 1 is dominant heterozygote and homozygote are in fact indistinguishable.

The establishment of a stable polymorphism is the theme of Fig. 5. In the simulation, the initial population was again a small-sized beak one, now immersed in an ecology with a broad distribution of edible seed sizes available, peaking at middle-sized seeds. The fitness function that represents this situation is

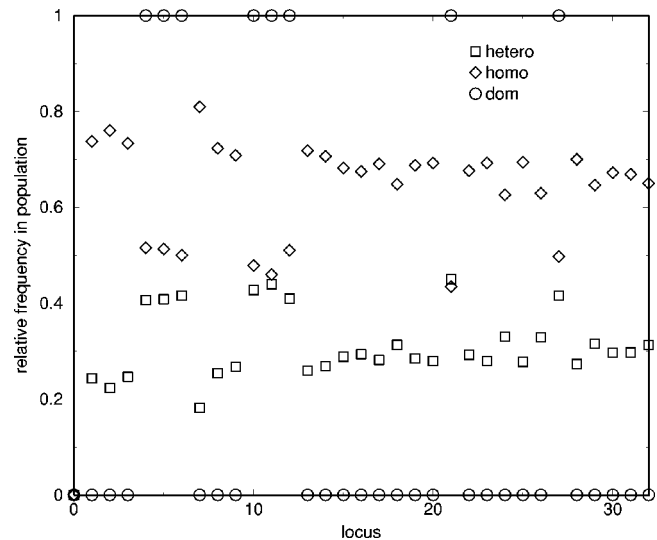


FIG. 4. Relative frequencies of homozygotes 11 and heterozygotes 10 in the equilibrium population. The circles indicate, for each locus, which is the dominant allele. This run had the same parameters as the ones from which Fig. 2 was extracted, except for the boost  $A=48$ .

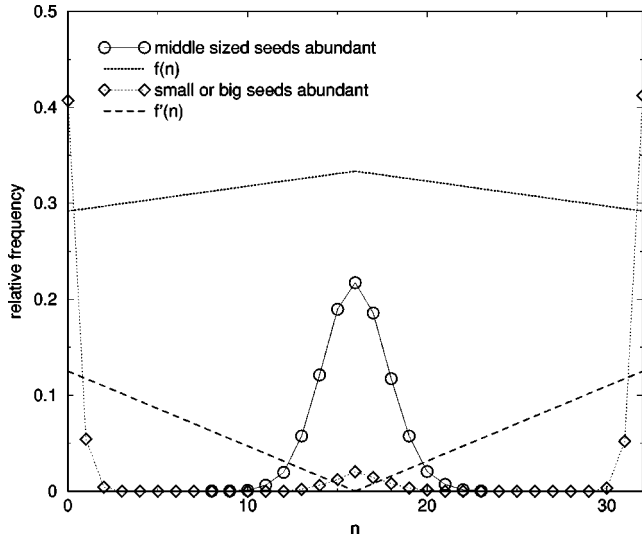


FIG. 5. The distribution of phenotypes in the population is shown for two different regimes of seed availability. The circles correspond to the equilibrium population in a situation in which seeds are available for a broad distribution of sizes, peaked at  $n = 16$ ; the corresponding fitness function  $f(n)$ , adequately rescaled to fit in the graph, is shown for a comparison. The two-peaked phenotype distribution corresponding to the diamonds sets in after a climate-induced change in the food supply alters the fitness function to  $f'(n)$ . The population splits into a (reversible) polymorphism, with different beak sizes. Since there is no reproductive isolation, mating between birds feeding on different niches generate offspring with medium-sized beaks, represented by the small bump at  $n = 16$ . The parameters of the simulation are the same as in the preceding figures, except where otherwise stated in the text.

$$f(n) = 1 - \frac{16-n}{A}, \quad n < 16$$

$$= 1 - \frac{n-16}{A}, \quad n \geq 16. \quad (3.2)$$

The population evolves for 20 000 Monte Carlo steps, when a snapshot of the phenotype distribution is taken. In agreement with the result shown in Fig. 2, this distribution is again bell shaped, with its peak now located at  $n = 16$ , corresponding to middle-sized beaks. Because mutations can both increase or decrease the beak size, as opposed to the case where the peak of the fitness function was at the largest possible extreme, and because the number of loci where each allele is dominant is the same, corresponding to a value  $d = 16$  for the dominance parameter, there is no inherent bias to the equilibrium distribution; its peak can sit at the same position as the one for the fitness function, as Fig. 5 shows. After 20 000 time steps, there is a sudden change in the pattern of seed availability, perhaps because of a variation in the rainfall regime caused by some global climate oscillation, whose effect is to decrease the availability of middle-sized seeds. The fitness function that expresses this new pattern is, for instance,

$$f'(n) = 1 - \frac{A-16+n}{A}, \quad n < 16$$

$$= 1 - \frac{A-n+16}{A}, \quad n \geq 16. \quad (3.3)$$

The evolution of the population resumes with this new fitness function, and with intraspecific competition within each niche. This is represented in the model by letting the mean field  $V(P)$  take into account *only the individuals that feed on a particular kind of seed*. Thus finches with small (large) beaks, represented by having  $n < (>) 16$ , and feeding on small (large) seeds, will compete only against those feeding on the same niche. To be specific, the simulation now computes at each time step  $P_m = \sum_{population} \theta(16-n)$  and  $P_M = \sum_{population} \theta(n-16)$ , where  $\theta(k)$  is the step function with  $\theta(0) = 1/2$ . To determine the probability of death by intraspecific competition for an individual, its beak size, which is proportional to the number  $n$  of 1 bits set in its phenotype, is used to decide in which niche it is competing. If  $n < (>) 16$ , its death probability will be  $V(P_m)/f'(n)[V(P_M)/f'(n)]$ . These elements alone force evolution to give rise to a polymorphism, shown in Fig. 5 as a resulting two-peaked equilibrium distribution of beak sizes. This polymorphism is reversible: if, in a subsequent time step, the pattern of availability of edible seeds reverts to its original configuration, so does the distribution of beak sizes. This is in complete agreement with the field observations of stable polymorphism in the Galapagos ground finches [9,10].

If such a stable polymorphism is not challenged by new climate variations for some time, it can induce speciation and generate two reproductively isolated segments, even without geographical barriers. To simulate this effect, in our model we introduced a gene that determines mating selection pattern. The initial population has a panmictic behavior, expressing no preferences related to the phenotype of the partner. When a female is ready to mate, she chooses a partner according to the expression of this gene. A randomly selected male in the population, to be accepted as a partner, has to either feed on the same niche, in which case the mating selection gene becomes irrelevant, or, if he feeds on a different niche, both parents have to be nonselective in their mating preferences for reproduction to occur. The offspring inherit the mating preferences of either the mother or the father, randomly selected at birth, and this gene can also suffer a mutation in either direction with some probability. The resulting distribution of this gene among the population is shown in Table I. After the double-peaked polymorphism is established, and if the distribution of edible seeds does not change for some time, reproductive isolation sets in.

#### IV. CONCLUSIONS

The addition of a multilocus nonstructured part to the age-structured genome of the Penna model, subjected to the same representation of evolution dynamics, allows a substantial broadening of its scope. The role of genes with age-independent effects can now be studied through the patterns

TABLE I. Fraction of the population with reproductive isolation. Mutations of mating preferences at birth occur with a probability  $p_{mate}=0.001$ . The results are averages over 10 runs.

|  | Males | Females |
|--|-------|---------|
| Small beaks and broad distribution of seeds      | 0.003 | 0.003   |
| Small beaks and two-peaked distribution of seeds | 0.939 | 0.940   |
| Large beaks and broad distribution of seeds      | 0.003 | 0.003   |
| Large beaks and two-peaked distribution of seeds | 0.952 | 0.954   |

that they generate in evolutionary populations. A particularly interesting side benefit is to dispose of the unmotivated random effect of the Verhulst dagger, and substitute it with interspecific competition with outcome based on the genetic load of the individuals. With the help of this broadened model, we studied the distribution of a single phenotypical trait among the population when it evolves on a single-peaked linear fitness landscape. The simulations exhibited its Gaussian-like character, with mean and width that depend in a clear way on the various parameters of the modified model. The distribution of homozygote and heterozygote loci of the nonstructured genome over the population also shows patterns that are easy to explain and to relate to the parameters. The present study opens the path for a variety of new applications of age-structured microscopic models, as it establishes their basic properties.

As a hint of the scope of these applications, we undertook a simulation of the effects of a varying pattern of food supply on the phenotype distribution. Our motivation were the reports from field observations of the variability of the beak morphology of the ground finch in the Galapagos Islands as a response to changes in seed availability due to seasonal

oscillations in rainfall regime. The results of the simulations show a striking parallelism with those observations, in that a polymorphism is generated as soon as the fitness landscape switches to a double-peaked pattern. Since modern theories on speciation seem to indicate that polymorphism is a necessary first step, we proceeded by introducing a gene that controls mating preferences. Mutation and evolution under a continued double-peaked fitness landscape were enough to generate reproductive isolation and speciation as a result. Speciation without geographic isolation has proven to be a difficult concept in evolution theory so far, but our simulations support a scenario in which its onset arises as a necessary evolutionary consequence of a continued double-peaked fitness landscape.

#### ACKNOWLEDGMENTS

We thank D. Stauffer for a critical reading of the manuscript. Research by J.S.S.M. was supported by CIRES, University of Colorado at Boulder. S.M.O. thanks the Brazilian agencies CNPq, CAPES, and FAPERJ for partial financial support.

- 
- [1] C. A. Tauber and M. J. Tauber, in *Speciation and Its Consequences*, edited by D. Otte and J. A. Endler (Sinauer Associates, Sunderland, MA, 1989).
  - [2] J. Roughgarden, *Theory of Population Genetics and Evolutionary Ecology: An Introduction* (Macmillan, New York, 1979).
  - [3] T. J. P. Penna, J. Stat. Phys. **78**, 1629 (1995).
  - [4] I. Mróz, A. Pekalski, and K. Sznajd-Weron, Phys. Rev. Lett. **76**, 3025 (1996); A. Pekalski, Physica A **252**, 325 (1998).
  - [5] J. Maynard Smith, Am. Nat. **100**, 637 (1966).
  - [6] J. Roughgarden, Am. Nat. **106**, 683 (1972); U. Dieckmann and M. Doebeli, Nature (London) **400**, 354 (1999).
  - [7] P. R. Grant and B. R. Grant, in *Speciation and Its Consequences* (Ref. [1]).
  - [8] P. T. Boag and P. R. Grant, Nature (London) **274**, 793 (1978); Science **214**, 82 (1981).
  - [9] P. R. Grant, *Ecology and Evolution of Darwin's Finches* (Princeton University Press, Princeton, 1986).
  - [10] D. Lack, *Darwin's Finches* (Cambridge University Press, Cambridge, 1983).
  - [11] S. Moss de Oliveira, D. Alves, and J. S. Sá Martins, Physica A **285**, 77 (2000).
  - [12] J. S. Sá Martins and S. Cebrat, Theory Biosci. **119**, 156 (2000).