Economics Letters 16 (1984) 177–184
North-Holland

MIGRATION, UNEMPLOYMENT AND DEVELOPMENT
A Dynamic Two-Sector Analysis *

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Received 6 December 1983

A time-minimizing problem of attaining a full employment state is solved in a dual-economy model with Harris–Todaro migration mechanism and with a positive level of urban unemployment in the starting period. It appears that the optimum solution lies in the specialization of investment in the urban sector at least in the initial stage of development if the per-capita capital stock in the urban sector is very small in the starting period.

1. Introduction

A static two-sector analysis on rural–urban migration and the urban unemployment problem, made by Todaro (1969,1976) and Harris and Todaro (1970) attempts to show that there is no strictly urban solution to the urban unemployment problem. The present paper shows that their conclusions may not be sustained when a dynamic analysis is made on the same problem even with the Harris–Todaro migration mechanism, particularly when the initial per-capita capital stock in the urban sector is at a very low level.

2. The model

The economy consists of an institutionally advanced urban sector and a backward rural sector. Both sectors produce the same commodity using

* This forms a part of chapter IV of my prospective Ph.D. dissertation. I am indebted to Professor A. Bose of I.I.M., Calcutta, for his valuable suggestions and comments. Remaining errors are solely mine.

capital and labour as inputs. Capital is measured in terms of the same commodity. The production function of the rural sector is of neo-classical type. But the urban sector has a fixed-coefficient production function.[1] The labour force required by the urban sector is supplied by the rural sector and the migration mechanism is of Harris–Todaro type. The planning authority of the advanced sector does not have any direct migration policy. It employs labour in the urban sector at an institutionally fixed wage-rate and allocates the surplus of the urban sector as investment between the two sectors. Capital once installed in either sector is non-shiftable. It does not depreciate over time. The rural wage-rate is equal to the average productivity of labour in the rural sector.[2] Wage-income in both sectors is fully consumed. The size of the labour-force grows at a constant rate.

Let 1 and 2 stand for urban and rural sector respectively. The notations consist of

k_i = capital stock in sector i as a ratio of total labour-force,
l_i = employment in sector i as a ratio of total labour-force,
l_u = urban unemployment as a ratio of total labour-force,
w = urban wage-rate,
x_2 = capital–labour ratio in sector 2,
f_2 = intensive production function of sector 2,
y_i = output in sector i as a ratio of total labour force,
λ = fraction of investment allocated to sector 1,
n = constant rate of growth in total labour force,
b_1 = technologically fixed capital–labour ratio in sector 1,
a = technologically fixed output–labour ratio in sector 1,
b_2 = capital–labour ratio in sector 2 required to produce w units of average productivity of labour in sector 2,
t = time,
i = 1,2.

[1] An analysis on this problem has been made in my prospective thesis even with a neo-classical production function in the urban sector.

[2] In a rural sector, characterized by family based peasant farming, it is an appropriate assumption.

The equational structure of the model is the following:

$$y_1 = a\,l_1 = (a/b_1)k_1 \tag{1}$$

$$y_2 = f_2(x_2)l_2, \tag{2}$$

$$\dot{k}_1 = ((\lambda/b_1)(a-w)-n)k_1, \tag{3}$$

$$\dot{k}_2 = ((1-\lambda)/b_1)(a-w)k_1 - nk_2, \tag{4}$$

$$l_u = 1 - l_1 - l_2, \tag{5}$$

$$(wl_1/(1-l_2)) = f_2(x_2). \tag{6}$$

The first five equations are familiar in the literature of growth theory. Eq. (6) is the Harris–Todaro migration equilibrium condition.

From the eqs. (5) and (6), one can get

$$l_u = ((w/f_2(x_2))-1)l_1. \tag{7}$$

It shows that $l_u \geqslant 0$ if $w \geqslant f_2(x_2)$. So there exists a positive level of urban unemployment in the presence of a rural–urban wage-gap even in migration-equilibrium. The economy is in full-employment when $w = f_2(x_2)$ or $l_1 = 1-(k_2/b_2)$

By definition, $l_1 = (k_1/b_1)$. So a necessary and sufficient condition for full-employment is given by

$$(k_1/b_1)+(k_2/b_2) = 1. \tag{8}$$

So it is $(k_1/b_1)+(k_2/b_2) < 1$ that makes full-employment impossible to attain at a particular period. Since k_1 and k_2 change only over time because of the non-shiftability of installed capital in both sectors, the economy can come out of unemployment state only at some future date. An important question in this dynamic model is: What should be the time-optimal investment policy if the full-employment state is to be attained in minimum time?

3. The dynamic optimization

The time-minimization problem of attaining a full-employment state is

$$\min \int_0^T \mathrm{d}t \quad \text{subject to} \quad (3), \quad (4), \quad (k_1(T)/b_1) + (k_2(T)/b_2) = 1,$$

$$(k_1(0)/b_1) + (k_2(0)/b_2) < 1 \quad \text{and} \quad 0 \leqslant \lambda \leqslant 1.$$

Here k_1 and k_2 are state variables and λ is a control variable. All these are functions of t.

It is assumed that $((a\text{-}w)/b_1) > n$. It is a necessary condition for the economy so that it can come out of the unemployment state at some future date.

The Hamiltonian is given by

$$H = -q_0 + q((a\text{-}w)/b_1)k_1 - nq_1k_1 - nq_2k_2,$$

where

$$q = \lambda q_1 + (1-\lambda)q_2.$$

Here q_0 is a constant and, with no loss of generality, one can assume $q_0 = 0$. q_1 and q_2 are functions of time. At each t, H is maximized by the choice of λ, given k_1, k_2, q_1 and q_2. The necessary optimality conditions [3] are

(i) $\lambda = 1$ if $q_1 > q_2$,
$0 \leqslant \lambda \leqslant 1$ if $q_1 = q_2$,
$\lambda = 0$ if $q_1 < q_2$.

(ii) There exist q_1, q_2, k_1, k_2, continuous functions of time, such that

$$\dot{k}_1 = ((\lambda/b_1)(a-w) - n)k_1,$$

$$\dot{k}_2 = ((1-\lambda)/b_1)(a-w)k_1 - nk_2,$$

$$\dot{q}_1 = nq_1 - q((a-w)/b_1), \tag{9}$$

$$\dot{q}_2 = nq_2. \tag{10}$$

[3] See Pontrjagin et al. (1962).

(iii) Transversality condition: $q_i \geqslant 0$ for $i = 1, 2$ with $q > 0$.

From (9) and (10), it is clear that

$$\dot{q}_2 - \dot{q}_1 = q((a-w)/b_1) + n(q_2 - q_1).$$

So $\dot{q}_2 > \dot{q}_1$ for $q_2 \geqslant q_1$.

Hence one can easily establish the following lemmas:

Lemma 1. *If $\lambda = 0$ is optimal for $t = t^*$, then $\lambda = 0$ is optimal for all $t \geqslant t^*$.*

Lemma 2. *There does not exist any non-degenerate interval $[t_0, t_1]$ such that $0 < \lambda < 1$ is optimal for all $t \in [t_0, t_1]$.*

So the optimal λ must have the following property:

$$\lambda = 1 \quad \text{for } 0 \leqslant t \leqslant \hat{t}, \quad \text{and}$$

$$\lambda = 0 \quad \text{for} \quad \hat{t} \leqslant t \leqslant T \quad \text{where} \quad 0 \leqslant \hat{t} \leqslant T.$$

Here, $\hat{t} = 0 \Rightarrow \lambda = 0$ for all t is optimal,
$\hat{t} = T \Rightarrow \lambda = 1$ for all t is optimal,
$0 < \hat{t} < T \Rightarrow$ A switch from $\lambda = 1$ to $\lambda = 0$ at some $\hat{t}$, $0 < \hat{t} < T$, is optimal.

Since for $0 \leqslant t \leqslant \hat{t}$, $\lambda = 1$ is optimal, one can get

$$\dot{k}_1 = (((a\text{-}w)/b_1) - n)k_1 \quad \text{and} \quad \dot{k}_2 = nk_2.$$

Hence the solutions are given by

$$k_1(\hat{t}) = k_1(0)\, \mathrm{e}^{(((a-w)/b_1)-n)\hat{t}}, \tag{11}$$

$$k_2(\hat{t}) = k_2(0)\, \mathrm{e}^{-n\hat{t}}. \tag{12}$$

For $\hat{t} \leqslant t \leqslant T$, $\lambda = 0$ is optimal. So one can get

$$\dot{k}_1 = -nk_1 \quad \text{and} \quad \dot{k}_2 = ((a-w)/b_1)k_1 - nk_2.$$

Hence the solutions are given by

$$k_1(T) = k_1(\hat{t})\,\mathrm{e}^{-n(T-\hat{t})}, \tag{13}$$

$$k_2(T) = \big(k_1(\hat{t})((a-w)/b_1)(T-\hat{t}) + k_2(\hat{t})\big)\,\mathrm{e}^{-n(T-\hat{t})}. \tag{14}$$

Now

$$(k_1(T)/b_1) + (k_2(T)/b_2) = 1$$

$$\Rightarrow b_1\big(k_1(0)\,\mathrm{e}^{((a-w)/b_1)\hat{t}}((a-w)/b_1)(T-\hat{t}) + k_2(0)\big)$$

$$+ b_2 k_1(0)\,\mathrm{e}^{((a-w)/b_1)\hat{t}} = b_1\, b_2\, \mathrm{e}^{nT}. \tag{15}$$

[Using eqs. (11), (12), (13) and (14).] eq. (15) shows T as a function of $\hat{t}$. T is to be minimized by an appropriate choice of $\hat{t}$ over the set $[0, T]$. Here,

$$(\mathrm{d}T/\mathrm{d}\hat{t}) = \frac{((a\text{-}w)/b_1)\,\mathrm{e}^{((a-w)/b_1)\hat{t}}k_1(0)\big(b_2 + b_1((T\text{-}\hat{t})((a\text{-}w)/b_1) - 1)\big)}{nb_1b_2\,\mathrm{e}^{nT} - b_1((a\text{-}w)/b_1)\,\mathrm{e}^{((a-w)/b_1)\hat{t}}k_1(0)}.$$

Now

$$(\mathrm{d}T/\mathrm{d}\hat{t}) = 0 \Rightarrow b_2 + b_1((T\text{-}\hat{t})((a\text{-}w)/b_1) - 1) = 0,$$

$$T\text{-}\hat{t} = (b_1 - b_2)/(a\text{-}w).$$

So $T \gtrless \hat{t}$ for $b \gtrless b_2$. Hence one can establish the following lemmas:

Lemma 3. *If $b_2 > b_1$, at some $\hat{t}$, $0 < \hat{t} < T$, T can not be minimum.*

Lemma 4. *If $b_2 < b_1$, T is minimum at some $\hat{t} < T$.*

If $\hat{t} = 0$, from (15) one can get

$$(k_1(0)/b_1) + (k_2(0)/b_2) + ((a\text{-}w)/b_1b_2)k_1(0)T = \mathrm{e}^{nT}.$$

At $T = 0$, left-hand side < 1 and right-hand side $= 1$. So at $T = 0$, it is not satisfied. So if a solution exists, $T > 0$ and one necessary condition

for the existence of solution is given by

$$((a\text{-}w)/b_1b_2)k_1(0)) > n \quad \text{or} \quad k_1(0) > (nb_1b_2/(a\text{-}w)).$$

So if $k_1(0) < (nb_1b_2/(a\text{-}w))$, there does not exist any $T > 0$, such that $(k_1(T)/b_1) + (k_2(T)/b_2) = 1$, when $\hat{t} = 0$. Hence one can establish

Lemma 5. *If $k_1(0) < (nb_2b_1/(a\text{-}w))$, T is minimized at some $\hat{t} > 0$.*

With the help of these, one can easily prove the following theorems.

Theorem 1. *If $k_1(0) < (nb_2b_1/(a\text{-}w))$ and $b_2 > b_1$, $\lambda = 1$ for all $t \geqslant 0$ is the optimal policy.*

Theorem 2. *If $k_1(0) < (nb_2b_1/(a\text{-}w))$ and $b_1 > b_2$, $\lambda = 1$ for all t, $0 \leqslant t \leqslant \hat{t} > 0$, and $\lambda = 0$ for all t, $0 \leqslant \hat{t} < t \leqslant T$, are the optimal policies.*

4. Conclusion

So the time minimization problem of attaining a full-employment state at the terminal period is basically an investment maximization problem in the urban sector at each period. The solution to the urban unemployment problem lies in the development of the urban sector at the most rapid rate when the urban sector is at a stage of underdevelopment. It does not mean that rural development is not important. The present model has not focused on the marketable surplus problem. A dynamic model of Dixit (1969), based on Lewis (1954) hypothesis, and focusing on the marketable surplus problem, has emphasized rural investment a great deal. Hence the present analysis demands a further dynamic analysis of the development problem of dual economies with adequate consideration to both urban unemployment and the marketable surplus problem.

References

Bose, S., 1968, Optimal growth and investment allocation, Review of Economic Studies, 465–480.

Bose, S., 1970, Optimal growth in a non-shiftable capital model, Econometrica.

Bhagwati, J.N. and T.N. Srinivasan, 1974, On re-analysing the Harris–Todaro model: Policy ranking in the case of sector specific sticky wages, American Economic Review, 502–508.

Dixit, A.K., 1968, Optimal development in the labour surplus economy, Review of Economic Studies, 23–24.

Dixit, A.K., 1969, Marketable surplus and dual development, Journal of Economic Theory, 203–219.

Harris, J.R. and M.P. Todaro, 1970, Migration, unemployment and development: A two-sector analysis, American Economic Review, 126–142.

Lewis, W.A., 1954, Economic development with unlimited supplies of labour, Manchester School, 139–191.

Pontrjagin, L.S. et al., 1962, The mathematical theory of optimal processes, Interscience.

Stern, N.H., 1972, Optimum development in a dual economy, Review of Economic Studies, 171–184.

Todaro, M.P., 1969, A model of labour-migration and urban unemployment in less developed countries, American Economic Review, 138–148.

Todaro, M.P., 1976, Urban job expansion, induced migration and rising unemployment, Journal of Development Economics, 211–225.