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Harris-Todaro Migration-Mechanism and the Optimum Development of the Urban Sector†

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ABSTRACT

A time-minimization problem of attaining a full-employment state is solved in a dual economy model where the rural-urban migration mechanism is of Harris-Todaro type. The optimum solution may appear as a policy of urban development at the most rapid rate.

1. INTRODUCTION

Rural-urban migration and urban unemployment problems have received considerable attention in the theoretical literature of development economics. The literature on this aspect starts with the models of Harris and Todaro (1970) and Todaro (1969, 1976) An institutionally given urban wage-rate and a wage-differential between the urban and the rural sectors form the basis of their framework. Migration from the rural sector to the urban sector results when actual rural wage-rate falls short of the expected urban wage rate defined as the actual urban wage rate multiplied by the ratio of urban employment to urban labour force. Migration equilibrium is established when expected urban wage-rate equals the actual rural wage rate; and the existence of urban-unemployment is explained as a migration-equilibrium phenomenon. One of the implications of those models is that the urban unemployment problem can not be solved by a policy of urban development.[1]

The basic Harris-Todaro (1970) model has been re-analysed and

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[1]See Todaro (1976 : 211-212, 226).

extended by various authors in various directions.[2] But neither the original Harris-Todaro (1970) model nor the other models focusing on urban unemployment problem are dynamic and intertemporal in nature and hence can not make capital-accumulation over time endogeneous to the analysis. The only dynamic model by Jha and Lachler (1981) examines the nature of optimum taxation on income of the various classes and the investment in the two sectors solving a Ramsey problem. But they do not focus on the minimum-time-solution to the urban unemployment problem. The other dynamic models of dual economies[3] are based on the Lewis (1954) hypothesis of perfectly elastic supply of labour from the rural sector to the urban sector at the fixed urban wage-rate, and hence they fail to focus on the urban unemployment problem. The objective of this paper is to develop a dynamic model of a less developed economy where one can study the theoretical properties of the optimum investment and employment policies in the urban sector in the context of the minimum-time solution to the urban unemployment problem. In order to establish consistency between migration-equilibrium and the existence of urban unemployment, the Lewis (1954) hypothesis is replaced by the Harris-Todaro (1970) migration-mechanism.

The model is described in Section 2 and a time-minimization problem of reaching a full-employment state is solved in Section 3. It appears that the optimum solution to the problem may be the development of the urban sector at the most rapid rate. This is somewhat different from the view of Todaro (1976) that there is no strict urban solution to the uaban unemployment problem.

2. THE MODEL

The economy consists of an institutionally advanced urban sector and a backward rural sector. The urban sector produces the product which is either consumed or used as fixed capital in both the sectors. The rural sector produces food. Both the sectors have CRS production-functions; and use capital and labour as inputs. The urban workers and the rural peasants consume food as well as the urban sector's product. Exchange between the urban workers and the peasants of the rural sector is assumed to be competitive.[4] The labour force required by the urban sector is supplied by the rural sector. There is no direct migration policy of the

[2]See, for example, Bhatia, (1979), Calvo (1978), Bhagwati and Srinivasan (1974).

[3]See, for example, Bardhan (1970), Dixit (1968, 1969), Stern (1972) and Marglin (1966, 1976).

[4]It is a simplifying assumption. Like Dixit (1968), one can consider the planner as the monopsonist in buying food and monopolist in selling those to the urban workers.

planning authority; and the migration-mechanism is of Harris-Todaro (1970) type. The planning authority employs labour in the urban sector at an institutionally fixed wage rate. Rural wage-rate is assumed to be equal to the average productivity of labour in rural sector.[5] Wage-income in both the sectors are consumed, and only the surplus of the urban sector is invested. The allocation of investment between the two sectors is subject to the control of the planninga uthority. Capital, once installed, is non-shiftable in either sector. Size of the labour-force of the economy grows over time at a constant rate. Capital once installed in either sector does not depreciate over time.

Let 1 and 2 stand for the urban and the rural sector. The notations consist of the followings :

k_i = Capital-stock in the ith sector, as a ratio of total labour force.

l_i = Employment in the ith sector, as a ratio of total labour force.

l_u = Urban unemployment as a ratio of total labour force.

x_i = Capital-labour ratio in the ith sector.

y_i = Average productivity of labour in the ith sector.

f_i = Intensive production function of the ith sector.

P = Price of rural sector's product in terms of the urban sector's product.

S_1 = Fraction of rural sector's output sold to the urban sector's workers.

S_2 = Fraction of urban wage income spent on the consumption of food.

W = Institutionally fixed urban wage rate.

λ = Fraction of investment allocated to the urban sector.

n = Constant rate of growth of labour force.

t = time.

T = Terminal period.

O = Starting period.

i = 1, 2.

[5]This is valid in a rural sector, characterized by family-based peasant farming.

The equational-structure of the model given by :

$$y_i = f_i(x_i) \quad \text{with} \quad f_i' > O \quad \text{and} \quad f_i'' < O \quad \text{for} \quad i = 1, 2 \tag{1}$$

is the intensive production function for the ith sector;

$$f_1(x_1) \geqslant w. \tag{2}$$

This implies that the surplus of the urban sector is non-negative. Output of the urban sector is the only source of financing the labour cost in that sector.

The two equations of motion, governing the behaviour of k_1 and k_2 over time, are :

$$\dot{k}_1 = \lambda(f_1(x_1) - w)\, l_1 - nk_1 \tag{3}$$

$$\dot{k}_2 = (1 - \lambda)\,(f_1(x_1) - w)\, l_1 - nk_2 \tag{4}$$

All these four equations are very much familiar in the literature of growth theory :

$$l_u = l - l_1 - l_2 \tag{5}$$

If $l_u > 0$, there is urban unemployment; and $l_u = 0$ implies that the economy is in full-employment, then

$$(Wl_1/(1 - l_2)) = f_2(x_2)\, P \tag{6}$$

is the migration equilibrium conditions of Harris and Todaro (1970). Here $(l_1/(1 - l_2))$ is the probability of obtaining an urban job of a rural migrant, $Wl_1/(1 - l_2))$ is his expected urban wage-rate and equation (6) implies that the actual rural wage rate in terms of the urban sector's product equals the expected urban wage-rate in migration equilibrium :

$$S_2(P_1) \cdot (W/P)\, l_1 = S_1(P)\, f_2(x_2)\, l_2, \tag{7}$$

where $S_1'(P) > O$;

$$O < S_1(P) < 1, \text{ for, } 1 < P < \infty;$$
$$S_2'(P) < O;$$

and, $O < S_2(P) < 1$, for $\infty > P > O$.

This equation implies that the demand for food from the urban sector's workers is equal to the supply of food to the urban sector. This is the equilibrium condition of competitive exchange between the urban sector and the rural sector.

At a particular point of time, k_1 and k_2 are given. Given W, the planning authority determines l_1. Now equations (6) and (7) determine l_2 and P; and equation (5) determines l_u. The planner determines λ, and hence $\dot{k}_1$ and $\dot{k}_2$ are determined. Thus one can get the values of k_1 and k_2 for the next point of time. Here λ and l_1 are subject to the control of the planner. Controlling the timepath of λ and l_1, it regulates the time-behaviour of k_1 and k_2. These ultimately determine the time-behaviour of l_u.

From equations (5) and (6), we have

$$l_u = ((W/P \cdot f_2(x_2)) - 1)l_1. \tag{8}$$

It shows that $l_u > O$ if $W > P \cdot f_2(x_2)$. So there exists a positive level of urban unemployment in the presence of rural-urban wage-gap. The economy is in full-employment when

$$W = Pf_2(x_2), \tag{9}$$

which means that the two wage-rates are equal.

Using equations (6) and (7), we find that

$$(S_1(P)/S_2(P)) = ((1 - l_2)/l_2) \tag{10}$$

We can not go further into the analysis without assuming any algebraic form of $S_1(P)$ and $(S_2(P)$. We assume that $S_1(P) = 1 - (1/P)$; and $S_2(P) = (1/P)$. Note that both these two algebraic forms satisfy the assumed properties of $S_1(P)$ and $S_2(P)$. Now from equation (10), we find,

$$((1 - (1/P))/(1/P)) = ((1 - l_2)/l_2),$$

$$\text{or,} \quad (1/P) = l_2. \tag{11}$$

So using equations (9) and (11), we find that the condition for full-employment is given by

$$W = (f_2(x_2)/l_2). \tag{12}$$

We assume that the production function of the rural sector is Cobb-Douglas, and is given by

$$f_2(x_2) = x_2^{\beta} \quad \text{with} \quad O < \beta < 1,$$

where β is the capital-elasticity of output in the rural sector. Hence from equation (12), we find that

$$l_2 = (1/b_2)\, k_2^{*} \tag{13}$$

is the condition for full-employment, where

$$b_2 = (W)^{(1/(1+\beta))}, \quad \text{and} \quad k_2^* = k_2^{(\beta/(1+\beta))}.$$

We define b_1 such that $f_1(x_1) =, W$, at $k_1 = b_1$. Hence b_1 is the capital-labour ratio in the urban sector required to yield W unit of average productivity of labour in that sector.

From inequality (2), we have

$$x_1 \geqslant b_1$$

$$\text{or.} \quad (k_1/b_1) \geqslant l_1. \tag{14}$$

In full-employment, $l_1 = 1 - l_2$. So it appears from equations (13) and (14) that a necessary and sufficient condition for full employment is given by

$$(k_1/b_1) + (k_2^*/b_2) \geqslant 1, \tag{15}$$

It is obviously necessary because if $(k_1/b_1) + (k_2^*/b_2) < 1$, then $l_1 = 1 - (k_2^*/b_2)$ implies $l_1 \geqslant (k_1/b_1)$ and hence contradicts (14). Similarly, it is sufficient also because if $(k_1/b_1) + (k_2^*/b_2) \geqslant 1$, then $l_1 = 1 - (k_2^*/b_2)$ clearly implies $(k_1/b_1) \geqslant l_1$. Hence we have the following proposition.

Proposition 1. If $(k_1/b_1) + (k_2^*/b_2) < 1$, the economy can not reach a full-employment state.

3. THE DYNAMIC OPTIMIZATION

If at the beginning of the plan, i.e. in period O, the historical parameters k_1 and k_2 (and hence, k_2^*), and the institutional constraint W are such that $(k_1(O)/b_1) + (k_2^*(O)/b_2) < 1$, then the economy can come out of the unemployment state only at some future date because k_1 and k_2 (and, hence k_2^*) change over time. Now an important problem for the planning authority is : what should be the employment policy in the urban sector and the investment-allocation policy between the rural and the urban sector if the full-employment state is to be attained in minimum time?

Using equation (4) and $k_2^* = k_2^{\beta/(1+\beta)}$, we find that

$$\dot{k}_2^* = (\beta/(1+\beta))\ k_2^*/k_2)\ \dot{k}_2$$

$$\text{or} \quad \dot{k}_2^* = (\beta/(1+\beta))\ (k_2^*)^{-(1/\beta))}\ ((1-\lambda)\ (f_1(x_1) - w)\ l_1 - nk_2)$$

$$\text{or} \quad \dot{k}_2^* = (1-\lambda)\ (f_1(x_1) - w)\ l_1\ (\beta/(1+\beta))\ (k_2^*)^{-(1/\beta))}$$
$$- (n\beta/(1+\beta))\ k_2^*. \tag{4.1}$$

The problem of attaining a full-employment state in minimum time is given by the following :
Minimize $\int_O^T dt$, subject to equations (3), (4.1);

$$(k_1(O)/b_1) + (k_2^*(O)/b_2) < 1;$$

$$(k_1(T)/b_1) + (k_2^*(T)/b_2) \geqslant 1;$$

$$O \leqslant \lambda \leqslant 1; \quad \text{and,} \quad O \leqslant l_1 \leqslant (k_1/b_1).$$

Here k_1 and k_2^* are state variables, and λ and l_1 are control variables All these are functions of time.

The Hamiltonian is given by

$$H = q_1\dot{k}_1 + q_2\dot{k}_2^*$$

or, $$H = q(f_1(x_1) - w)\, l_1 - nq_1k_1 - nq_2\, k_2^*\, (\beta/(1 + \beta)), \tag{16}$$

where, $q = \lambda q_1 + (1 - \lambda)\, q_2\, (\beta\,(1 + \beta))\, (k^*)^{-(1/\beta)}$, q_1 and q_2 being two co-state variables, functions of time

The Hamilton-Lagrange is defined as

$$HL = H + h((k_1/b_1) - l_1)$$

At each t, HL is maximized by the appropriate choice of λ and l_1. Here h is the Lagrange-multiplier.

The necessary optimality conditions[6] are :

(i) $q(f_1(x_1) - f_1'(x_1)\, x_1 - w) - h = O$

where $h \geqslant O$ if $l_1 \leqslant (k_1/b_1)$

(ii) Optimal $\lambda \begin{Bmatrix} = 1 \\ \in [0, 1] \\ = 0 \end{Bmatrix}$ if $q_1 \begin{Bmatrix} \gtreqless \\ < \end{Bmatrix} q_2((\beta/(1 + \beta))\, (k_2^*)^{-(1/\beta)}$.

(iii) The dual variables, q_1 and q_2 satisfy the following :

$$\dot{q}_1 = -(\partial H/\partial k_1) - h, \tag{17}$$

$$\dot{q}_2 = -(\partial H/\partial k_2^*). \tag{18}$$

(iv) Transversality condition: $q_i \geqslant 0$ for $i = 1, 2$; $q > 0$; $(q_1(T), q_2(T))$ is orthogonal to $[k_1(T)/b_1) + (k_2^*(T)/b_2) = 1$.

[6]See Pontriagin *et al.* (1962).

If $l_1 = (k_1/b_1)$ is optimal, then the optimum time-path of k_i satisfies $\dot{k}_i = -nk_i < 0$ for $i = 1, 2$. So optimum

$$k_i = k_i(O)\, e^{-nt} \quad \text{for} \quad i = 1, 2.$$

Hence, optimum $k_i \to O$ as $t \to \infty$. for $i = 1, 2$. Note that

$$k_2^* = k_2^{(\beta/(1+\beta))}.$$

So if $k_2 \to O$ as $t \to \infty$, then

$$k_2^* \to O \text{ as } t \to \infty.$$

So if

$$(k_1(O)/b_1) + (k_2^*(O)/b_2) < 1,$$

then there does not exist any T, $O < T < \infty$, such that

$$(k_1(T)/b_1) + (k_2^*(T)/b_2) \geq 1$$

for optimum k_1 and k_2^*. So optimum $l_1 < (k_1/b_1)$ and hence $h = 0$. Now from optimality condition (i), we have

$$q(f_1(x_1) - f_1'(x_1)\, x_1 - W) = 0$$

From the transversality condition, we have, $q > 0$ So,

$$f_1(x_1) - f_1'(x_1)\, x_1 = W.$$

This implies that the marginal productivity of labour equals to the wage rate in the urban sector which is the condition of surplus maximization in the urban sector.

Hence one can easily establish the following proposition:

Proposition 2. The optimal choice of technique in the urban sector is surplus-maximizing.

We define g_1 such that $f_1(x_1) - f_1'(x_1)\, x_1 = W$ at $x_1 = g_1$. So g_1 is the surplus-maximizing capital-labour ratio in the urban sector.

We also define a such that $f_1'(g_1) = a$, So a is the marginal productivity of capital in the urban sector with surplus-maximizing technique. It is also assumed that, $a > n$. This implies that the rate of capital-accumulation is higher than the rate of population-growth.[7]

[7]It is a necessary condition for the economy so that it can come out of unemployment trap at least at some future date.

Using Proposition 2 which implies $x_1 = g_1$, and the definition, $f_1'(g_1) = a$, we find from equation (16) that

$$H = qak_1 - nq_1k_1 - nq_2k_2^* \, (\beta/(1 + \beta)). \tag{19}$$

Since optimum $h = 0$, using equations (17) and (19), we have,

$$\dot{q}_1 = nq_1 - qa. \tag{20}$$

Using equations (18) and (19), we have

$$\dot{q}_2 = n(\beta/(1 + \beta))\, q_2 - ak_1 \, (\partial q/\partial k_2^*)$$

or, $$\dot{q}_2 = n(\beta/(1 + \beta))\, q_2 + ak_1 (1 - \lambda)\, q_2(1/\beta)\, (k_2^*)^{-((1+\beta)/\beta)}(\beta/(1 + \beta)). \tag{21}$$

Since $x_1 = g_1$ is optimal, and $f_1'(g_1) = a$, from equations (3) and (4.1) we have

$$\dot{k}_1 = ak_1 - nk_1. \tag{22}$$

and

$$\dot{k}_2^* = (1 - \lambda)\, ak_1\, (\beta/(1 - \beta))\, (k_2^*)^{-(1/\beta)} - (n\beta/(1 + \beta))\, k_2^* . \tag{23}$$

Can an interior value of λ ever be optimal? This is possible if

$$q_1 = q_2(\beta/(1 + \beta))\, (k_2^*)^{-(1/\beta)} = q$$

and

$$\dot{q}_1 = \dot{q}_2(\beta/(1 + \beta))\, (k_2^*)^{-(1/\beta)} - q_2\, (\beta/(1 + \beta))\, (1/\beta)\, (k_2^*)^{-((1+\beta)/\beta)}\, \dot{k}_2^*. \tag{24}$$

Now using equations (21) and (23), we have

$$\text{R.H.S. of (24)} = n(\beta/(1 + \beta))\, q_2(k_2^*)^{-(1/\beta)} > 0.$$

If $q_1 = (\beta/(1 + \beta))\, q_2\, (k_2^*)^{-(1/\beta)} = q$, then

$$\text{R.H.S. of (24)} = nq_1 > 0.$$

Since $a > n$, (by assumption) we find from equation (20) that

$$\text{L.H.S. of (24)} = \dot{q}_1 = q_1(n - a) < 0.$$

So the equation (24) is never satisfied. Hence we have the following proposition.

Proposition 3. Interior investment allocation can not be optimal.
If $\lambda = 0$ is optimal, then

$$q = q_2(\beta/(1+\beta))\,(k_2^*)^{-(1/\beta)} > q_1,$$

R.H.S. of (24) $= nq > 0$; and the L.H.S. of (24) $= \dot{q}_1 = nq_1 - qa < 0$ because $n < a$; and $q_1 < q$.

Note that $\lambda = 0$ is optimal if

$$q_1 < q_2(\beta/(1+\beta))\,(k_2^*)^{-(1/\beta)}.$$

Also now we find that if $\lambda = 0$ is optimal, then

$$\dot{q}_1 < \dot{q}_2\,(\beta/(1+\beta))\,(k_2^*)^{-(1/\beta)} - q_2(\beta/(1+\beta))\,(1/\beta)\,(k_2^*)^{-((1+\beta)/\beta)}\,\dot{k}_2^*$$

So we can prove that if

$$q_1 < q_2\,(\beta/(1+\beta))\,(k_2^*)^{-(1/\beta)}$$

at some $t = t^*$, then

$$q_1 < q_2\,(\beta/(1+\beta))\,(k_2^*)^{-(1/\beta)}$$

for all $t \geqslant t^*$. In other words, if $\lambda = 0$ is optimal at $t = t^*$, then $\lambda = 0$ is optimal for all $t > t^*$.
Hence we can prove the following proposition.

Proposition 4. Any switch from specialization of investment to the rural sector to that to the urban sector can not be optimal.

So the optimum investment-allocation policy must be a specialization either to the urban sector alone or to the rural sector alone or to the urban sector in the initial stage followed by that to the rural sector in the terminal stage. Mathematically optimum λ must satisfy the following property :

$$\lambda = 1 \text{ for } O \leqslant t < \hat{t}, \quad \text{and}$$
$$\lambda = O \text{ for } \hat{t} \leqslant t < T,$$

where $O \leqslant \hat{t} \leqslant T$. Here,

$\hat{t} = O$ implies that $\lambda = O$ is optimal for all $t \geqslant 0$;

$\hat{t} = T$ implies that $\lambda = 1$ is optimal for all $t \geqslant 0$;

and $O < \hat{t} < T$ implies that a switch from $\lambda = 1$ to $\lambda = O$ at some $\hat{t}$, $O < \hat{t} < T$, is optimal.

Since for $O \leqslant t \leqslant \hat{t}$, $\lambda = 1$ is optimal, in this phase, we have, from equations (22) and (23),

$$\dot{k}_1 = (a - n)\, k_1, \quad \text{and}$$

$$\dot{k}_2^* = -\,(n\beta/(1 + \beta))\, k_2^*$$

The solutions are given by the following :

$$k_1\,(\hat{t}) = k_1\,(0) \cdot e^{(a-n)\hat{t}}, \text{ and} \tag{25}$$

$$k^*\,(\hat{t}) = k_2^*\,(0) \cdot e^{-(n\beta/(1+\beta))\hat{t}} \tag{26}$$

For $\hat{t} \leqslant t \leqslant T$, $\lambda = O$ is optimal. So in this phase, from equation (22), we have,

$$\dot{k}_1 = -\,nk_1$$

and its solution is given by

$$k_1(T) = k_1(\hat{t}) \cdot e^{-n(T-\hat{t})} \tag{27}$$

But for $\lambda = O$, from equation (4), we find that

$$\dot{k}_2 = (f_1(x_1) - w)\, l_1 - nk_2,$$

and using Proposition 2 and the definition, $f_1'(g_1) = a$, we find that

$$\dot{k}_2 = ak_1 - nk_2$$

and its solution is given by

$$k_2(T) = (k_1(\hat{t})\, a(T - \hat{t}) + k_2(\hat{t}))\, e^{-n(T-\hat{t})}$$

We know that $k_2^* = k_2{}^{(\beta/(1+\beta))}$. Therefore,

$$k_2^*(T) = (k_1(\hat{t})\, a(T - \hat{t}) + k_2(\hat{t}))^{(\beta/(1+\beta))}\, e^{-(n\beta/(1+\beta))(T-\hat{t})}. \tag{28}$$

Since for $0 \leqslant t \leqslant \hat{t}$, $\lambda = 1$ is optimal, we have

$$\dot{k}_2 = -\,nk_2, \text{ and}$$

$$k_2(\hat{t}) = k_2(O) \cdot e^{-n\hat{t}} \tag{29}$$

Now using equations (25) and (27), we have

$$k_1(T) = k_1(0)\, e^{a\hat{t}}\, e^{-nT}, \tag{30}$$

and using equations (25), (28) and (29), we have

$$k_2^*(T) = (k_1(O) \cdot e^{a\hat{t}}\, a(T - \hat{t}) + k_2(0))^{(\beta/(1+\beta))}\, e^{-n(\beta/(1+\beta))T}. \tag{31}$$

Now $l_1 = (k_1/g_1)$ is optimal where g_1 is the surplus maximizing technique; and from equation (13), we have, $l_1 = 1 - (k_2^*/b_2)$ is the condition of full-employment. So the full-employment condition consistent with surplus maximizing choice of technique is given by

$$(k_1(T)/g_1) + (k_2^*(T)/b_2 = 1. \tag{32}$$

Note that $g_1 > b_1$. So for given k_1 and k''_2,

$$(k_1/g_1) + (k_2^*/b_2) < (k_1/b_1) + k_2^*/b_2).$$

So, if some (k_1, k_2^*) satisfies condition (32), then (k_1, k_2^*) must satisfy (15). So condition (32) is a sufficient condition for full-employment.

Using equations (30), (31) and (32), we have

$$b_2 k_1(O)\, e^{a\hat{t}} + g_1(k_1(O)\, e^{a\hat{t}}\, a(T - \hat{t}) + k_2(O))^{(\beta/(1+\beta))}\, e^{(a/(1+\beta))T} = g_1 b_2\, e^{aT} \tag{33}$$

Equation (33) shows that T is a function of $\hat{t}$. So T is to be minimized by an appropriate choice of $\hat{t}$ over the set $[0, T]$. But the functional relationship is highly complicated and it is not becoming possible to find out optimum $\hat{t}$ evaluating $(dT/d\hat{t})$. So, we follow some different lines of analysis.

Note that $\hat{t} > O$ implies that $\lambda = 1$ is optimal for all $t \in [O, t]$; and for all t in this time-interval,

$$q_1 > q_2\, (\beta/(1 + \beta))\, (k_2^*)^{-(1/\beta)}.$$

For, $\lambda = 1$, solving the differential equations (20) and (21) over the time interval $[O, \hat{t}]$, we have

$$q_1(\hat{t}) = q_1(O) \cdot e^{(n-a)\hat{t}}, \tag{34}$$

$$q_2(\hat{t}) = q_2(O) \cdot e^{(n\beta/(1+\beta))\hat{t}}. \tag{35}$$

Now using equations (35) and (26), we have, at $t = \hat{t}$,

$$q_2(\beta/(1 + \beta))\, (k_2^*)^{-(1/\beta)} = q_2(O) \cdot (k_2^*(O))^{-(1/\beta)} \cdot e^{n\hat{t}}. \tag{36}$$

Here $t = \hat{t}$ is a switch point. So at $t = \hat{t}$, we have,

$$q_1 = q_2(\beta/(1 + \beta))\, (k_2^*)^{-(1/\beta)}. \tag{37}$$

Now using equations (34), (36) and (37) we have

$$\hat{t} = (1/a)\ \log\ ((q_1(O)/q_2(O))\, (k_2^*(O))^{(1/\beta)}). \tag{38}$$

Here, $\hat{t} > O$ if $q_1(O) > q_2(O) \cdot (k_2^*(O))^{-(1/\beta)}$.

Now using Proposition 4, we can prove that if

$$q_1(O) \leqslant q_2(O)\,(k^*(O))^{-(1/\beta)},$$

then $\lambda = O$ for all $t \geqslant O$ is optimal. This leads to the following proposition :

Proposition 5. If, a policy of specialization of investment to the rural sector is optimal,

$$q_1(O) \leqslant q_2(O)\,(k_2^*(O))^{-(1/\beta)},$$

But if

$$q_1(O) > q_2(O))\,(k_2^*(O))^{-(1/\beta)},$$

then optimum $\hat{t}$ is either positive but less than T, or is equal to T. What happens if $\hat{t} = T$? For, $\hat{t} = T$, from equation (33), we have,

$$1 + (g_1 k^*(O)/b_2 k_1(O))\, e^{((n/(1+\beta))-a)T} = (g_1/k_1(O))\, e^{(n-a)T},$$

$$\text{or, } (g_1 k_2^*(O)/b_2 k_1(O))\, e^{((n/(1+\beta))-a)T} < (g_1/k_1(O))\, e^{(n-a)T},$$

$$\text{or, } (1/n\beta/(1+\beta)))\, \log\, (k_2^*(O)/b_2) < T. \tag{39}$$

Hence from (38) and (39), it is clear that, $\hat{t} < T$, if

$$(1/a)\ \log\ ((q_1(O)/q_2(O))\,(k^*(O))^{(1/\beta)}) \leqslant (1/(n\beta/(1+\beta))) \log\,(k_2^*(O)/b) \tag{40}$$

Hence we can prove the following proposition.

Proposition 6. If condition (40) is satisfied, and if

$$q_1(O) > q_2(O)\,(k_2^*(O))^{-(1/\beta)},$$

then a switch from specialization of investment to the urban sector to that to the rural sector at some intermediate time-point is optimal.

If condition (40) is not satisfied, and if

$$q_1(O) > q_2(O)\,(k_2^*(O))^{-(1/\beta)},$$

Then $\hat{t}$ may be equal to T. So $\lambda = 1$ for all $t \in [O, T]$ may be optimal. This leads us to prove the following proposition.

Proposition 7. If condition (40) is not satisfied, and if

$$q_1(O) > q_2(O)\,(k_2^*(O))^{-(1/\beta)},$$

Then a policy of specialization of investment to the urban sector may be optimal.

4. CONCLUSION

What we have analyzed is a two sector dynamic plan model of a less developed economy with an objective of eliminating the urban unemployment problem in minimum time. We have adopted the Harris-Todaro migration mechanism. But what is interesting is that the optimum solution to this problem does not necessarily support the view of Harris and Todaro that there is no strict urban solution to the urban unemployment problem and the solution lies in the rural development. Obviously like any other dynamic optimizing models, the optimum solutions in this model depends on the initial conditions. But the initial conditions may alter the nature of the game. If $k_2^*(O)$, $q_1(O)$ and $q_2(O)$ are such that

$$q_1(O) > (k_2^*(O))^{-(1/\beta)}\, q_2(O),$$

Then the optimum solution to the problem of eliminating urban unemployment in minimum time lies in the development of the urban sector at the most rapid rate at least in the initial stage of the development programme. The programme of urban development and the programme of solving urban unemployment problem are not always contradictory to each others.

Why do we get a result which, to some extent, goes against the view of Harris-Todaro? This is because we have adopted a dynamic framework to analyse the problem. In a dynamic model, allocation of investment and the accumulation of capital are endogeneous to the analysis. Also in this model, surplus of the urban sector is the only source of investment. So the development of the urban sector enlarges the capacity of generating surplus (investment) and creates employment opportunities in that sector. On the otherhand, additional employment in the urban sector raises the level of demand for food and hence the price of food. This process eliminates the rural-urban wage gap at some finite time-point. It is well-known that Harris and Todaro have adopted a static one period general equilibrium model which can not make capital-accumulation over time endogeneous to the analysis.

What we get from the model is known as a bang-bang solution : specialize in the urban sector or the rural sector or specialize in the urban sector for a while and then switch completely to the rural sector. Interior investment allocation does not appear to be optimum at all. This bang-bang solution is the result of the following two assumptions : (i) Surplus of the urban sector is the only source of investment, and (ii) the wage rate of the urban sector is institutionally fixed in terms of the urban sector's product. So the Hamiltonian that is maximized at each point of time is a positive function of the urban sector's capital stock, but not of the capital stock of the rural sector. But if it is assumed that the urban sector's wage rate is fixed in terms of food, then capital-accumulation in the rural sector will lower the price of food and hence the industrial wage-bill. In this case, capital accumulation in the rural sector will have a positive effect on the level of investment (urban sector's surplus). It will be optimal to invest in both the sectors simultaneously if the marginal contribution of each sector's capital-stock on the urban sector's surplus is equal. Dixit (1969) develops a planning model of a less developed dual economy and solves a time-minimization problem of urban development in the context of marketable surplus problem faced by the urban sector. He assumes that the urban sector's wage rate is fixed in terms of food and finds that optimum solution lies in the policy of balanced growth of both the sectors.

Note that we assume $S_1'(P) > O$ in this model. But the opposite assumption about the sign of $S_1(P)$ is also reasonable. If we assume that $S_1(P) = P^{-Z}$, then from equation (10), we have,

$$(1 + P^{1-Z}) = (1/l_2)$$

And hence the inverse relationship between P and l_2 remains unchanged so long $Z < 1$. Hence, using equation (12), we find that, in full employment, $l_2 = l_2(k_2^*)$ with $l_2' > O$. Now from equation (14), it is clear that $(k_1/b_1) + l_2(k_2^*) \geqslant 1$ is a necessary and sufficient condition for full-employment. The nature of the time-optimal solution does not change so long $l_2' > O$. So even if $S_1'(P) < O$, but the absolute value of price-elasticity of supply of food, Z, is less than unity, the optimum solution to the problem does not change.

Obviously the model is abstract and fails to consider the role of different types of rural and urban institutions like trade unions, share tenancy, money-lenders etc. on the rural-urban migration. But the static analysis of Harris and Todaro has also the same limitations. On the other hand a dynamic analysis is better than a static one when the former can make accumulation of capital and the allocation of investment endogeneous to the model, and is expected to be of some interest when it opposes the conclusions of a static analysis. At this stage, we should not say that the results of Harris and Todaro are trivial or unimportant. But the present analysis clearly points out the importance of any further analysis on migration and urban uemployment problem in a more general dynamic model.

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